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THE PUPILS' ARITHMETIC
BOOK FIVE



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TORONTO

THE PUPILS' ARITHMETIC

BOOK FIVE

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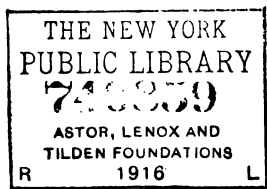
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PREFACE

BOOK FIVE of *The Pupils' Arithmetic* contains a thorough and comprehensive treatment of all the topics usually taught in the seventh year. Percentage in its various relations and with special emphasis upon its practical applications, simple interest including the indirect cases together with the various methods of finding interest, bank discount, bills and notes are all fully and clearly presented. This volume also includes an extended treatment of business forms, simple accounts, foreign moneys, the metric system, and an introduction to algebra, including the solution of simple equations and of arithmetical problems by algebraic methods.

The appendix of this volume contains a list of the tables used in connection with the study of mensuration, practical measurements, and the metric system.

The development of the work in this volume is in accordance with the general plan of the series.

The special features are :

- I. Reading and discussion as an introductory study to new topics.
- II. The clear indication of the steps in progress in the treatment of each topic.

III. The use of type forms and of model solutions in problem-work.

IV. Numerous graded exercises for oral and written practice, and an abundance of real problems from everyday life for thoughtful consideration.

V. Summaries and reviews.

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THE PUPILS' ARITHMETIC
BOOK FIVE

PUPILS' ARITHMETIC

BOOK V

NUMERATION AND NOTATION

1. Notation is the writing of numbers by means of characters; *e.g.* eighty-one is written 81 or LXXXI.

2. Numeration is the naming or reading of numbers written by means of characters; *e.g.* 81 is read eighty-one.

ARABIC OR DECIMAL NOTATION

The common mode of writing numbers is called the **Arabic notation**. It was first introduced into Europe by the Mohammedan Arabs, who had acquired it from the Hindus of India. It has been in general use among Europeans for about four hundred years. It is sometimes called the **decimal system of notation**.

3. The Arabic or decimal notation is distinguished by four features:

(1) **Ten numerals, figures, or digits are used as characters; i.e.** 1, 2, 3, 4, 5, 6, 7, 8, 9, 0.

The denominator of a decimal fraction is the name of the order to the right of the decimal point occupied by the last figure of the numerator; *e.g.* .025 is read 25 thousandths, the numerator being 25 and the denominator being thousandths.

The removal of a decimal figure one place to the left multiplies its value by 10, and its removal one place to the right divides its value by ten; *e.g.* .40 multiplied by 10 becomes 4.0; and 0.5 divided by 10 becomes .05.

NOTE. Moving the decimal point to the right produces the same result as moving the figure to the left; and moving the decimal point to the left produces the same result as moving the figure to the right.

6. A **mixed decimal** is a whole number and a decimal fraction written together.

7. A **complex decimal** is a decimal containing a common fraction; *e.g.* $.5\frac{3}{4}$ is read five and three fourths tenths; $.0785\frac{1}{3}$ is read seven hundred eighty-five and one third ten-thousandths.

EXERCISES

8. Read aloud:

- | | | | |
|----------|-----------|------------|-------------|
| 1. .1234 | 2. .12345 | 3. .123456 | 4. 200.0020 |
| .7068 | .20640 | .260450 | 4060.00065 |
| .9600 | .03090 | .100900 | 900.000900 |
| .0745 | .00781 | .084010 | 84.00084 |
| .0109 | .42008 | .100604 | 27.0087 |

Read the same numbers, after moving the decimal points one place to the right. Two places. Three places.

Read the fractions given in the next exercises.

WRITTEN EXERCISES

9. Write decimally :

- | | | | |
|-----------------------|------------------------|-------------------------|------------------------|
| 1. $\frac{56}{10000}$ | 2. $\frac{703}{10000}$ | 3. $\frac{2064}{10000}$ | 4. $\frac{86}{100000}$ |
| $\frac{450}{100000}$ | $\frac{703}{1000000}$ | $\frac{2064}{10}$ | $\frac{7}{10000}$ |
| $\frac{906}{1000000}$ | $\frac{703}{10}$ | $\frac{2064}{1000000}$ | $\frac{3285}{10000}$ |
| $\frac{2400}{100000}$ | $\frac{703}{100000}$ | $\frac{2064}{1000}$ | $\frac{7060}{100000}$ |

Seventy hundred-thousandths; seven ten-thousandths; seven hundred millionths; seven hundred hundred-thousandths.

Miscellaneous Examples in Reduction of Decimals

SIGHT EXERCISES

10. 1. Reduce to tenths : .60 ; .400 ; .7000
 2. Reduce to hundredths : .4 ; .3 ; .7 ; $.3\frac{1}{2}$
 3. Reduce to highest decimal order : .400 ; .35000 ; .016000 ; .20700 ; .30090
 4. Reduce to thousandths : .9 ; $.8\frac{1}{4}$; .0325
 5. Change to hundredths : .060 ; .0100 ; .755
 6. Change to thousandths : .1500 ; .68 ; .25000
 7. Change to ten-thousandths : .07560 ; .08500
 8. Change to hundred-thousandths : .7506200

WRITTEN EXERCISES

11. Reduce to common fractions or to mixed numbers in lowest terms :

- | | | |
|--------------------------|------------------------|-------------------------|
| 1. .0325 | 2. 1.62500 | 3. 8.0720 |
| 4. 18.1875 | 5. .24 | 6. .0400 |
| 7. $6.194\frac{4}{9}$ | 8. $5.11\frac{1}{2}$ | 9. $6.0066\frac{2}{3}$ |
| 10. $12.0042\frac{2}{5}$ | 11. $1.014\frac{2}{7}$ | 12. $10.016\frac{2}{3}$ |
| 13. $.02\frac{1}{2}$ | 14. $.01\frac{1}{4}$ | 15. $.428\frac{4}{7}$ |

EXERCISES

12. Reduce to decimal form :

- | | | | |
|---------------------|------------------------|-----------------------|----------------------|
| 1. $\frac{81}{125}$ | 2. $\frac{16}{25}$ | 3. $\frac{41}{50}$ | 4. $\frac{7}{25}$ |
| 5. $\frac{9}{16}$ | 6. $\frac{11}{16}$ | 7. $\frac{56}{125}$ | 8. $\frac{18}{70}$ |
| 9. $7\frac{4}{60}$ | 10. $14\frac{51}{120}$ | 11. $25\frac{1}{80}$ | 12. $66\frac{1}{60}$ |
| 13. $\frac{7}{800}$ | 14. $\frac{7}{8000}$ | 15. $\frac{7}{80000}$ | 16. $25\frac{1}{32}$ |
17. Compare: .487 with $\frac{5}{12}$; .387 with $\frac{9}{25}$.
 18. Which is greater, .287 or $\frac{2}{7}$? .435 or $\frac{4}{9}$?
 19. Which is less, $\frac{84}{125}$ or .6385? 1.167 or $1\frac{1}{6}$?

FUNDAMENTAL PROCESSES

13. The fundamental processes of arithmetic are addition, subtraction, multiplication, and division.

14. Like numbers are numbers that consist of units of the same kind. 2 pounds and 3 pounds are like numbers.

ADDITION**Types of Oral Drills for Frequent Practice**

- 15.** 1. Add by 9's to the number nearest 100. What is the number?
2. Add by 7's from 3 to the first total beyond 100. What is that total?
3. Beginning with 6, add 9 at each tap of the pencil.
4. Find the sum of: 41, 8, 8, 9, 12, 5, 9, 14.
5. What is the amount of 75 ct., 18 ct., 19 ct., 37 ct., 15 ct.?
6. Add 15 to each of the following: 18, 21, 35, 27, 39, 43, 56.
7. Add 39 and 22; 59 and 22; 79 and 22.
8. Add 28 and 15; 38 and 15; 48 and 15.

SUBTRACTION**Types of Oral Drills for Frequent Practice**

- 16.** 1. Beginning with 100, subtract 7 at each tap of the pencil. Give the remainder.
2. How much must be added to \$2.71 to make \$5.80?
3. Beginning with 95, subtract 8 until a remainder ending with the figure 5 is reached. Write your answer.

4. Subtract 9 from each of the following: 51, 42, 55, 67, 83, 94, 38.

5. I had \$4.50. I spent \$1.30, 25 ct., 20 ct., 18 ct., 22 ct. How much had I left?

Find the change from a one-dollar bill of the following amounts:

6. 18 ct. 7. 47 ct. 8. 57 ct. 9. 65 ct. 10. 76 ct.

From a two-dollar bill:

11. \$1.44	12. \$1.56	13. \$1.13	14. \$.87
15. \$.19	16. \$1.39	17. \$1.61	18. \$.54

MULTIPLICATION

Types of Oral Drills for Frequent Use

17. 1. Add by 15's to 100.

2. Write the twelfth multiple of 3; of 6; of 9; of 12; of 5; of 10; of 7; of 4; of 8; of 11.

3. How much is 11×8 ? 5×13 ? 4×18 ?

4. How many are 3×15 ? 8×15 ? 16×15 ?

5. How many are 10 times 38? 5 times 38?

6. $3 \times 17 = 51$; $6 \times 17 = ?$ $9 \times 17 = ?$

7. $(5 \times 8) + (6 \times 8) = ?$ $(18 \times 5) - (3 \times 5) = ?$

8. $(4 \times 15) + (4 \times 15) = ?$ $(5 \times 16) + (5 \times 16) = ?$

9. $(9 \times 15) - (4 \times 15) = ?$ $(12 \times 16) - (4 \times 16) = ?$

10. How much is 15×15 ? 20×20 ?

DIVISION

Types of Oral Drills for Frequent Use

18. 1. How much is $72 \div 12$; by 3; by 4; by 6?
2. $550 \div 10 = ?$ $1100 \div 10 = ?$ $2150 \div 10 = ?$
3. One factor of 245 is 5; what is the other factor?
4. Give the sets of factors of 24; 35; 36; 40.
5. How many 6's in 36; in 54; in 60; in 96?
6. $? \div 3 = 12$; $? \div 4 = 12$; $? \div 6 = 12$?
7. Add the quotients of $48 \div 4$ and $28 \div 7$.
8. Go from *A* to *B* as quickly as possible, and then back from *B* to *A*.

Divide by 7; by 21.



9. Begin with 100; multiply by 4; divide by 50; multiply by 9; subtract 8; divide by 16; add 37.

WRITTEN EXERCISES

19. Add, looking for combinations of 10.

Verify each answer by adding in reverse order.

Read the numbers aloud and then add silently.

Verify each answer as in example 1.

1. 16497	2. 53754	3. 3497016
8534	71895	8205767
7625	108407	14087128
908	7659	9453342
20654	33792	27908136
18998	89525	40075000
3941	875000	7745845
7075	98876	6078974
<u>42</u>	729302	3892474
39	10701	20967845
48	78844	3807049
39	204721	10007006
4	68973	30800049
<u>84232</u>		

Add vertically and horizontally and compare the totals.

4. \$ 256.84	\$ 416.83	\$ 125.75
318.19	219.44	264.82
420.08	377.25	194.38
294.65	240.80	227.55
301.98	329.65	203.46
<u>301.98</u>	<u>329.65</u>	<u>203.46</u>

ANALYSIS OF PROBLEMS

Forms of statement and forms of analysis in the solution of problems in multiplication and division by integers.

20. The complete solution of a problem contains (1) the statement, (2) the analysis, (3) the computation (or algorism), and, possibly, (4) the proof or verification. Some solutions may properly be accompanied by a diagram.

1. (Multiplication.) At \$6.25 per barrel, how much will 9 bbl. of flour cost ?

Statement	Analysis
1 bbl. \$ 6.25	The cost of 9 bbl. is 9 times the cost of 1 barrel. Therefore, 9 bbl. will cost $9 \times \$ 6.25$, or \$56.25. <i>Ans.</i>
9 bbl. ?	

2. (Multiplication.) How many collars can be bought for \$15, if 8 collars cost \$1 ?

Statement	Analysis
\$ 1 8 col.	\$ 15 will buy 15 times as much as \$1. Therefore, \$15 will buy 15×8 collars, or 120 collars. <i>Ans.</i>
\$ 15 ?	

NOTE TO TEACHERS. Pupils should be trained to set down problems in a clear but brief form. The statements presented herewith make conspicuous the **correspondence** (*i.e.* the ratios) of

the given terms and the required term. The required term, indicated by ? or x , is preferably placed last. Mere correspondence is indicated by the (. . . .), and not by the sign of equality (=).

3. (Partition.) If 16 yd. of cloth cost \$40, what is the cost of 1 yd. ?

Statement	Analysis
16 yd. \$40	The cost of 1 yd. is $\frac{1}{16}$ of the cost of 16 yd. Therefore, the cost of 1 yd. is $\frac{1}{16}$ of \$40 = \$2.50. <i>Ans.</i>
1 yd. ?	

4. (Partition.) If \$18 will buy 144 handkerchiefs, how many will \$1 buy ?

Statement	Analysis
\$18 144 h.	\$1 will buy $\frac{1}{18}$ as many as \$18. Therefore, \$1 will buy $\frac{1}{18}$ of 144 handkerchiefs, or 8 handkerchiefs. <i>Ans.</i>
\$1 ?	

5. (Division.) At 75 ct. per yard, how many yards may be bought for \$15?

Statement	Analysis
\$.75 1 yd.	The number of yards will equal the number of times that 75 ct. is contained in \$15. Therefore, the number of yards is $1500 \div 75$ ($15 \div \frac{3}{4}$), or 20 yd. <i>Ans.</i>
\$15 ?	

6. (Partition and Multiplication.) If 24 books cost \$18, how much will 100 books cost?

Statement	Analysis
24 b. \$18	Use Form 3 to find the cost of 1 book and then Form 1 to find the cost of 100 books.
100 b. ?	

7. If the third term (100) in example 6 is changed to some number that is an *exact multiple* of the first term (24), Form 1 may be used to advantage, *e.g.*:

If 24 books cost \$18, how much will 96 books cost?

Statement	Analysis
24 b. \$18	96 books will cost 4 times as much as 24 books. Therefore, 96 books will cost $4 \times \$18$, or \$72. <i>Ans.</i>
96 b. ?	

8. If the third term (100) in example 6 is changed to some number that is a *factor* of the first term (24), then Form 3 used to advantage, *e.g.*:

If 24 books cost \$18, how much will 4 books cost?

Statement	Analysis
24 b. \$18	The cost of 4 books is $\frac{1}{6}$ of the cost of 24 books. Therefore, 4 books will cost $\frac{1}{6}$ of \$18, or \$3. <i>Ans.</i>
4 b. ?	

9. (Partition and Division.) If 25 yd. cost \$7.50, how many yards can be bought for \$69?

Statement	Analysis
\$ 7.50 25 yd.	Use Form 3 to find the cost of 1 yd. and then Form 5 to find the required number.
\$ 69 ?	

10. If the third term (\$69) in example 9 is changed to some number that is a multiple of the first term (\$7.50), Form 2 may be used to advantage, *e.g.* :

If 25 yd. cost \$750, how many yards can be bought for \$75?

Statement	Analysis
\$7.50 25 yd.	\$75 will buy 10 times as much as \$7.50. Therefore, \$75 will buy 10×25 yd., or 250 yd. <i>Ans.</i>
\$75 ?	

11. If the third term (\$69) in example 9 is changed to a number that is a factor of the first term (\$7.50), then Form 4 may be used to advantage, *e.g.* :

If 25 yd. cost \$7.50, how many yards can be bought for \$2.50?

Statement	Analysis
\$7.50 25 yd.	\$2.50 will buy $\frac{1}{3}$ as much as \$7.50. Therefore, \$2.50 will buy $\frac{1}{3}$ of 25 yd., or $8\frac{1}{3}$ yd. <i>Ans.</i>
\$2.50 ?	

WRITTEN PROBLEMS

21. Give statement and analysis of each of the following :

GIVEN		TO FIND	
QUANTITY BOUGHT	TOTAL COST	COST OF	HOW MUCH FOR
1. 18 gal.	\$ 27.00	1 gal.	—
2. 40 rings	\$ 140.00	10 rings	—
3. 25 yd.	\$ 162.50	—	\$ 300.00
4. 1 A.	\$ 335.00	25 A.	—
5. 72 hats	\$ 118.00	144 hats	—
6. 27 doz.	\$ 90.00	9 doz.	—
7. 48 books	\$ 36.00	1 book	—
8. 120 sq. ft.	\$ 15.00	—	\$ 62.50
9. 24 pkg.	\$ 1.00	—	\$ 5.00
10. 18 qt.	\$ 1.75	50 qt.	—
11. 35 lb.	\$ 75.00	25 lb.	—
12. 150 boxes	\$ 22.50	50 boxes	—
13. 16 yd.	\$ 1.20	—	\$ 6.00
14. 32 collars	\$ 4.00	—	\$ 1.00
15. 20 chairs	\$ 125.00	—	\$ 50.00
16. 1 shirt	\$ 1.75	—	\$ 56.00
17. 15 M. bricks	\$ 112.50	100 M.	—
18. 3 pr. hose	\$ 1.00	—	\$ 15.00
19. 1 cap	\$.88	24	—
20. 60 lamps	\$ 24.00	—	\$ 96.00
21. 3000 bd. ft.	\$ 75.00	80 M. bd.ft.	—
22. 50 Kg.	\$ 60.00	—	\$ 300.00
23. 600 M.	\$ 720.00	150 M.	—
24. 12 Km.	\$ 96.00	96 Kg.	—
25. 120 l.	\$ 24.00	—	\$ 3.00
26. 60 Ha.	\$ 600.00	10 Ha.	—
27. 45 gro.	\$ 60.00	—	\$ 200.00

MISCELLANEOUS PROBLEMS

22. 1. A cashier began a day's business with \$45.73 on hand. His cash receipts during the day were as follows:

\$3.50	\$2.14	\$5.47	\$16.75
2.50	7.67	6.25	8.24
.75	10.55	5.10	9.38
7.25	3.29	.48	5.05
6.00	6.78	1.50	17.49
1.38	.89	6.49	21.16

He paid out the following amounts: postage, \$ 5.50; wages, \$ 15.00; gas bill, \$ 17.80; wrapping-paper, \$ 8.20.

Find the cash balance.

2. A merchant's bank balance on Feb. 1 was \$ 14,957. His deposits during the month were: Feb. 4, \$ 367.40; Feb. 6, \$ 459.82; Feb. 9, \$ 314.70; Feb. 11, \$ 187.67; Feb. 13, \$ 234.15; Feb. 16, \$ 542.68; Feb. 20, \$ 816.00; Feb. 23, \$ 208.90; Feb. 25, \$ 477.69; Feb. 27, \$ 718.70.

He drew checks on the bank as follows: Feb. 2, \$ 425.00; Feb. 8, \$ 584.50; Feb. 10, \$ 250.00; Feb. 16, \$ 1980.72; Feb. 23, \$ 2679.29; Feb. 27, \$ 5167.47. Find his bank balance of March 1.

3. A stenographer earning \$ 1000 per year computed her expenses for next year as follows:

board \$ 450, clothing \$ 250, vacation \$ 50, lunches and carfare \$ 105, miscellaneous expenses \$ 100. How much may she save?

4. If 25 yd. of silk cost \$ 52, how many yards can be bought for \$ 75 ?

5. How much must I pay for 8000 copies of a book at the rate of 16 for \$ 1 ?

6. What will 150 gross of pens cost at 75 ct. per gross ?

7. If bricks are worth \$ 12.50 per thousand, what will 200,000 bricks cost ?

8. A certain grade of men's hose is retailed at the price of \$ 1 for 3 pr. Find the cost of 2120 pr.

9. At 40 ct. a dozen, what is the price of 50 crates of eggs, each crate containing 30 doz. ?

10. If 18 handkerchiefs cost \$ 5.40, what will 25 cost ?

11. How long will 12 bbl. of flour supply a company of 98 men, allowing 2 lb. per man a day ?

12. The areas of the public lands of the United States reported in 1911 were: Surveyed, one hundred eighty-eight million, eight hundred eighty-nine thousand, one hundred thirty-six acres; Un-surveyed, six hundred ninety-five million, four hundred one thousand, two hundred fifty-nine acres. Write the total number of acres in words.

13. A commission merchant sold for A. Vale: 260 lb. of butter at 36 ct., 320 lb. of cheese at 30 ct., 300 doz. eggs at 35 ct., and 720 lb. chicken at 23 ct. His charges were: freight \$ 21.60, storage \$ 2.75, cartage \$ 6.20, commission \$ 23.01. How much should he remit to Vale?

14. A speculator bought a building for \$ 26,500, spent \$ 2150 in altering the building, and sold it for \$ 31,250. How much was his profit?

15. A tailor bought 6 pieces of goods containing 42 yd. each; he used 38 yd. of the first piece, 39 yd. of the second, 35 yd. of the third, 37 yd. of the fourth, 41 yd. of the fifth, and 23 yd. of the sixth.

(a) How many yards did he have left in the remnants?

(b) He received \$ 1.75 per yard for the goods sold to customers, and 90 ct. per yard for the remnants. If the goods cost \$ 1.25 per yard, how much did he gain by the entire transaction?

16. A commercial traveler purchased a thousand-mile ticket and used it on the following trips: 315 mi., 48 mi., 96 mi., 28 mi., 47 mi., 116 mi., 237 mi., 14 mi., 29 mi., 56 mi. How much mileage had he left out of the thousand-mile ticket?

17. The mileage book cost \$ 20. The regular fares for the trips mentioned were respectively: \$ 9.10, \$ 1.50, \$ 3, \$.90, \$ 1.35, \$ 3.50, \$ 8, \$.35,

\$.85, \$ 1.60. How much was saved by using the mileage book?

18. A traveling salesman sent the following expense account to his firm for one week: railroad fares \$ 19.72, hotel bills \$ 18, telegraph and telephone \$ 7.60, excess baggage \$ 9.80, carriage hire \$ 2.50, sundries \$ 6.50. Find the total amount.

19. A freight car has a capacity 32,500 lb.; a second one has a capacity of half as much more. What is the joint capacity of the two cars?

20. A "local train" is moving along a railroad track at a uniform speed of 23 mi. an hour; an "express train" moves along a parallel track in the same direction at a uniform speed of 62 mi. per hour. How far ahead will the "express train" be 4 hr. and 15 min. after it passes the "local train"?

ADDITION AND SUBTRACTION OF DECIMALS

RULE. To add or subtract decimals, write like orders under one another and proceed as with whole numbers.

EXERCISES

23. Find sums:

1. 3.098; 42.2706; 108.5; 907.08067; 16.09
2. 250.0786; 2.1909; 749.009; 841.0176
3. 2078.7; 170.07; 200.02; 747.700; 264.0981
4. 56204.341; 9.0007; 281.403; 1010
5. 808.00095; 6.872354; .298017; .6731429

6. 27.0702; 5600; .08040; 18; 47.3987
7. 94.72; 94567.09; 1200; 365; 40.0008

EXERCISES

24. Find remainders:

- | | |
|-----------------------------------|------------------------------|
| 1. $267.018 - 148.84$ | 4. $47.0035 - 25.764$ |
| 2. $26.17 - 18.049$ | 5. $941.076 - 279.4091$ |
| 3. $2000.10 - 1476.0804$ | 6. $2846.5 - 1009.60754$ |
| 7. $6.6 - .002578$ | 8. $90,675.207 - 84,072.004$ |
| 9. $2,000,000 - 1,284,750.68$ | |
| 10. $4,619,584.57 - 3,984,631.89$ | |

EXERCISES

25. Find results, correct to thousandths:

- | | |
|---|---|
| 1. $25\frac{3}{8} + 16.08 - 3\frac{2}{5}$ | 2. $18\frac{3}{4} - 16.075 + 9\frac{1}{3}$ |
| 3. $106 - 41\frac{5}{6} + 57.018$ | 4. $14\frac{1}{7} + 21\frac{1}{2} - 5.021$ |
| 5. $200 - 74\frac{3}{4} - 26.0475$ | 6. $20.175 + 56\frac{1}{9} + 45\frac{2}{3}$ |
| 7. $65\frac{7}{10} - 42.625 + 6$ | 8. $16\frac{5}{16} + 35\frac{5}{12} - 42.025$ |

EXERCISES

26. Find results according to the following method:

(1) Add to the first number all the other numbers that are preceded by the plus sign. (2) Add together all the numbers preceded by the minus sign. (3) Subtract the second sum from the first sum.

- | |
|--|
| 1. $24.18 + 281.374 - 106.87 - 48.2089 + 17.3$ |
| 2. $64.289 - 14.75 + 28.333 + 71.556 - 18.35$ |
| 3. $9.095 + 28.385 - 17.066 - 41.833 + 75.625$ |

MULTIPLICATION

27. RULE. To multiply one decimal by another, multiply as with whole numbers and point off as many places from the right of the product as the sum of the decimal places in multiplicand and multiplier.

NOTES. 1. Since a whole number may be regarded as a decimal number containing 0 decimal places, the same rule applies to the multiplication of a whole number and a decimal; *e.g.* $8.478 \times 52 = 440.856$, since $(3 + 0)$ decimal places = 3.

2. In pointing off, it is sometimes necessary to prefix ciphers to the product in order to get the proper number of decimal places; *e.g.* $.012 \times .06 = .00072$.

EXERCISES

Find products :

- | | |
|-----------------------------|---------------------------|
| 1. $32.5 \times .75$ | 2. 45.62×14.7 |
| 3. 167.084×45 | 4. 17.058×2.009 |
| 5. $2000 \times .0625$ | 6. 4.50×2.080604 |
| 7. 7894.008×35.64 | 8. 16.900×725 |
| 9. 605.40×27.00748 | |

To multiply by a Whole Number ending in One or More Ciphers

Multiply 25.625 by 80; by 800; by 80,000.

$$25.625 \times 80 = 256.25 \quad \times 8 = 2050.$$

$$25.625 \times 800 = 2562.5 \quad \times 8 = 20,500.$$

$$25.625 \times 80,000 = 256,250 \quad \times 8 = 2,050,000.$$

RULE. To multiply a decimal by a whole number ending in one or more ciphers, move the decimal

point in the multiplicand as many places to the right as there are ciphers in the multiplier and multiply by the significant * figures of the multiplier.

EXERCISES

28. 1. Multiply by 20: 16.50; .081; 3.7; 156
2. Multiply by 600: 35.76; 28.925; 9.367
3. Multiply by 4000: 2.5; 76.03; .087
4. Multiply by 90,000: 5.6; 37.50; 1.218
5. Multiply by 47,000: 9.5; 46.13; 7.485
6. Multiply by 8,000,000: 7.8; 12.63; 17.086
7. Multiply by 325,000: 6.3; 66.33; 9.111

A SHORT METHOD WITH MULTIPLIERS EQUIVALENT TO
EASY COMMON FRACTIONS

Multiply 42.15 by .25; by .625; by $.83\frac{1}{3}$

$$42.15 \times .25 = 42.15 \times \frac{1}{4} = 10.5375$$

$$42.15 \times .625 = 42.15 \times \frac{5}{8} = \frac{210.75}{8} = 26.35625$$

$$42.15 \times .83\frac{1}{3} = 42.15 \times \frac{5}{6} = \frac{210.75}{6} = 35.125$$

NOTE. Time may sometimes be saved by reducing a decimal multiplier to the equivalent common fraction.

* The significant figures of a number are the figures that remain after the ciphers adjacent to the decimal point are struck out; *e.g.* in 20,500 the significant figures are 2, 0, and 5.

SIGHT EXERCISES

29. 1. Multiply by .50, giving answers at once:

166	2.72	1.980	.8248
4.36	28.50	2.742	.82568

2. Multiply by .25:

160	328	6.4	.1728
7.20	2.564	24.368	.47960

3. Multiply by .125:

8000	160	2.4	.0144
------	-----	-----	-------

4. Multiply by $.33\frac{1}{3}$:

900	1.50	9.6	.06246
2.34	28.176	125.073	.957612

MISCELLANEOUS WRITTEN EXERCISES

30. 1. How much is $6\frac{1}{4}$ hundredths of \$ 3200 ?
 2. Find .125 of \$ 24.30
 3. What is $.83\frac{1}{3}$ of \$ 72.50 ?
 4. Multiply 3.0385 by 250.8
 5. Solve. $.33\frac{1}{3}$ of 28.45×16.00845
 6. What is the product of $\frac{5}{6}$ of 1.008 by 291 ?
 7. How much is 25 thousandths of 2.325 times \$ 5.145 ?
 8. Multiply 8 ten-millionths by 12 thousandths
 9. Find: .75 of 150 times 12.025 ft.
 10. How much is 4.85 lb. multiplied by $.06\frac{1}{4}$?

DIVISION

31. RULE. To divide by a decimal.

1. Change the divisor to a whole number by moving the decimal point to the right of the last figure.

2. Move the decimal point in the dividend as many places to the right as the decimal point was moved in the divisor; if necessary, annex ciphers to the dividend.

3. Divide as with whole numbers.

4. Place the decimal point in the quotient directly above the decimal point in the dividend.

WRITTEN EXERCISES

32. Divide:

1. $.923$ by $.671$

2. 8.89 by 1.49

3. $.864$ by 1235

4. 108.4 by 63.13

5. 19.3666 by 12.84

6. 31.94 by 6.104

7. 51323 by $.1315$

8. 693.45 by 8.412

9. 89.61 by 1.108

10. 395.16 by 4.8015

EXERCISES

33. 1. $12.76 \div 784.009 = ?$ 2. $.34 \div .7286 = ?$

3. $213 \div 1.8 = ?$

4. $18.104 \div 8.40 = ?$

5. $8.615 \div .68 = ?$

6. $16.1406 \div 1.16 = ?$

7. $87.48 \div 8.10 = ?$

8. $41.738 \div 9.04 = ?$

9. $3.75 \div .62572 = ?$

10. $.0012 \div 7.25 = ?$

EXERCISES

34. Find the quotients:

1. $1.36\frac{1}{2} \div .714$

2. $78.1\frac{1}{10} \div 41.1\frac{1}{2}$

3. $67.3\frac{1}{5} \div 7.834$

4. $.341\frac{1}{4} \div 7.85\frac{1}{2}$

5. $49.6 \div 3.41$

6. $78.9 \div 4.31$

7. $3.81\frac{1}{2} \div 19.8\frac{1}{5}$

8. $84\frac{1}{2} \div 78.1\frac{1}{10}$

9. $63.41\frac{1}{4} \div 76\frac{1}{8}$

10. $17.48 \div 376.54$

**Division by a Whole Number ending in One or More
Ciphers**

Divide 78.096 by 60; by 600; by 60,000.

$$78.096 \div 60 = 7.8096 \div 6 = 1.3016.$$

$$78.096 \div 600 = .78096 \div 6 = .13016.$$

$$78.096 \div 60,000 = .0078096 \div 6 = .0013016.$$

RULE. To divide a decimal by a whole number ending in one or more ciphers, move the decimal point in the dividend as many places to the left as there are ciphers in the divisor and divide by the significant figures of the divisor.

EXERCISES

35. 1. Divide by 70: 14.42; 6.3; .287; .0098

2. Divide by 900: 18.9; 6.309; 75.15; .018

3. Divide by 8000: 456.8; 29.04; 7.008; .056

4. Divide by 40,000: 2868.72; 3.12; 96.88

5. Divide by 300,000: 6000; 18.51; 8.463

6. Divide by 5,000,000: 1000; 7.25; .35

7. Divide by 25,000: 3.6; .185; .7; .0009

Division by a Decimal Equivalent to an Easy Common Fraction

36. Divide 3.016 by .5; by .25; by $.66\frac{2}{3}$

$$3.016 \div .5 = 3.016 \div \frac{1}{2} = 3.016 \times 2 = 6.032.$$

$$3.016 \div .25 = 3.016 \div \frac{1}{4} = 3.016 \times 4 = 12.064.$$

$$1.508$$

$$3.016 \div .66\frac{2}{3} = 3.016 \div \frac{2}{3} = 3.016 \times \frac{3}{2} = 4.524.$$

Time may sometimes be saved by reducing a decimal divisor to the equivalent common fraction.

EXERCISES

37. Find quotients:

1. $41.3 \div .5$

2. $.125 \div .25$

3. $17.2 \div .33\frac{1}{3}$

4. $2.5 \div .83\frac{1}{3}$

5. $24.75 \div .75$

6. $.48 \div .66\frac{2}{3}$

7. $.72 \div .125$

8. $.15 \div .375$

9. $60 \div .625$

10. $.14 \div .875$

11. $1.2 \div .14\frac{2}{7}$

12. $.25 \div .025$

13. $11 \div .025$

14. $.9 \div .08\frac{1}{3}$

15. $.07 \div .16\frac{2}{3}$

16. $4.1 \div .11\frac{1}{9}$

MISCELLANEOUS WRITTEN EXERCISES

38. 1. How much is 96 ten-thousandths divided by 3 tenths?

2. One factor of .1805 is .025. What is the other factor?

3. Solve: $2.0016 \div .54$

4. What is the quotient of 4.8 divided by .000076?

5. Solve: $(21.6 + .034) \div (1.2 \times .018)$
6. How much is $.025$ of $.33\frac{1}{3} \times 7.5$?
7. Solve: $1.5(2.04 - .0068) + .83\frac{1}{3}$ of 2.16
8. Solve: $\frac{3\frac{1}{2}}{15}$ times $.018 \div 7.2$
9. Solve: $2.5(.098 + 1.272) - .66\frac{2}{3}$ of 1.5
10. Solve: $.08(.004 - .0025) + \frac{2}{5}$ of $.20$

MISCELLANEOUS PROBLEMS IN DECIMALS

39. 1. 950 T. of coal were shipped to New York. $.72$ of it was hard coal, the rest was soft coal. How many tons of each kind were there?

2. A manufacturer bought 250 T. of coal. He used $.52$ of it.

(a) How many tons were used?

(b) How many tons were not used?

3. A book which cost \$ 2.50 was sold at a gain of $.45$ of its cost.

(a) Find the gain.

(b) Find the selling price.

4. A wagon which cost \$ 125 was sold at a loss of $.24$ of the cost.

(a) Find the loss.

(b) Find the selling price.

5. 1500 ft. is what decimal part of 1 mi. (5280 ft.)?

6. 1750 lb. is what decimal part of a ton?

7. A grocer had 240 doz. oranges. He sold 200 doz. What part of the total amount (expressed in decimals) was sold? What part was not sold?

8. An automobile marked at \$ 1800 was reduced to \$ 1375. What reduction (expressed decimally) was made in the price of the automobile?

9. The distance from New York to Pittsburgh is 440.5 mi. The distance between New York and Philadelphia is .208 of that distance.

(a) How far is it from New York to Philadelphia?

(b) How far is it from Philadelphia to Pittsburgh?

10. A steamboat that is making a trip of 875 mi. has gone 675 mi. What part of the entire distance (expressed decimally) has been covered?

11. .37 of the height of the Woolworth Building, New York; is 277.5 ft. How high is the building?

12. The population of a city in 1910 was .21 greater than the population in 1900. The actual increase was 147,000 persons. Find the population in 1900, and the population in 1910.

13. .095 of the amount of money received for an entertainment is \$ 261.25. What was the total amount received?

14. 500 bbl. of apples were sent to a wholesale dealer. .46 of them were sold at \$ 2.50 per barrel. The rest were sold at \$ 2.75 per barrel. How much was received for all the apples?

15. The wages of the employees in a factory were increased .12. The manufacturer paid \$ 636.06 more than formerly on account of the increase. Find the total amount formerly paid for wages.

16. The population of Boston is 670,500; the population of Massachusetts is 3,366,000. The population of Boston is what decimal part of the population of Massachusetts?

17. A horse is sold at a gain of .28 of its cost. The gain is \$ 175. Find the cost.

18. A number of boxes of lemons weighed 1352 lb. .02 of the weight was due to the wooden boxes. Find the actual weight of the lemons; of the boxes.

19. A nickel (5-cent piece) is made of copper and nickel. 3 parts are copper and 1 part is nickel. What part of the coin (express as a decimal fraction) is copper?

20. $24\frac{1}{2}$ lb. is what part (express as a decimal fraction) of 196 lb.?

21. If a boat travels .33 of its journey in 39.6 hr., how many hours are required for the entire journey?

COMMON FRACTIONS

40. All operations with fractions depend upon the following important principles:

1. A fraction is an indicated division with the dividend above the line and the divisor below it; *e.g.* $\frac{12}{3} = 2 \div 3$.

2. Multiplying or dividing both terms of a fraction by the same number does not change the value of the fraction.

These principles are also expressed by the fundamental axiom: $1 = \frac{1}{1} = \frac{2}{2} = \frac{3}{3} = \frac{4}{4} = \frac{100}{100} = \frac{20}{20}$.

REDUCTION OF FRACTIONS

EXERCISES

41. 1. How many tenths in 5 units? In 8 units? In 10?

2. How many fifths in $4\frac{2}{5}$? In $6\frac{1}{5}$? In $10\frac{3}{5}$?

Reduce to improper fractions:

3. $12\frac{5}{6}$ $11\frac{8}{9}$ $12\frac{4}{7}$ $6\frac{1}{4}$ $12\frac{3}{5}$

4. $16\frac{2}{3}$ $10\frac{9}{10}$ $18\frac{3}{4}$ $26\frac{2}{3}$ $15\frac{5}{8}$

5. How many units in 65 twelfths? In 40 sixths?

Reduce to whole or mixed numbers :

6. $\frac{56}{9}$ $\frac{90}{7}$ $\frac{120}{9}$ $\frac{75}{8}$ $\frac{60}{11}$

7. $\frac{45}{8}$ $\frac{33}{5}$ $\frac{28}{3}$ $\frac{41}{4}$ $\frac{50}{6}$

8. How many 24th in $\frac{2}{3}$? In $\frac{3}{4}$? $\frac{5}{6}$? $\frac{3}{8}$? $\frac{7}{8}$?

9. How many 48ths in $\frac{1}{2}$? In $\frac{2}{3}$? $\frac{5}{6}$? $\frac{7}{8}$? $\frac{7}{12}$?

10. Reduce $\frac{5}{8}$ to 40th. To 72ds. To 80ths.
To 200ths.

Reduce to lowest terms :

11. $\frac{20}{24}$ $\frac{48}{60}$ $\frac{80}{120}$ $\frac{75}{90}$ $\frac{144}{180}$

12. $\frac{15}{24}$ $\frac{25}{60}$ $\frac{36}{60}$ $\frac{18}{24}$ $\frac{72}{128}$

Reduce to the least common denominator :

13. $\frac{3}{4}, \frac{2}{3}$ 14. $\frac{5}{6}, \frac{3}{4}$ 15. $\frac{3}{8}, \frac{7}{10}$

16. $\frac{4}{5}, \frac{9}{20}$ 17. $\frac{5}{8}, \frac{5}{12}$ 18. $\frac{5}{9}, \frac{1}{6}, \frac{3}{4}$

19. $\frac{2}{5}, \frac{9}{10}, \frac{1}{2}$ 20. $\frac{15}{24}, \frac{11}{18}$ 21. $\frac{4}{7}, \frac{2}{3}, \frac{1}{6}$

22. $\frac{4}{11}, \frac{2}{3}$ 23. $\frac{7}{10}, \frac{7}{12}$ 24. $\frac{1}{6}, \frac{1}{5}, \frac{1}{4}$

Reduce to simple fractions :

25. $\frac{1}{2}$ of $\frac{5}{8}$ 26. $\frac{2}{3}$ of $\frac{9}{10}$ 27. $\frac{3}{5}$ of $\frac{8}{9}$

28. $\frac{3}{4}$ of $2\frac{2}{3}$ 29. $\frac{4}{5}$ of $1\frac{5}{8}$ 30. $\frac{7}{9}$ of $5\frac{2}{3}$

31. $\frac{3}{8}$ of $4\frac{4}{15}$ 32. $\frac{4}{9}$ of $8\frac{1}{10}$

33. 34. 35. 36. 37.

Simplify: $\frac{3\frac{1}{3}}{5}$ $\frac{2\frac{2}{3}}{8}$ $\frac{1\frac{1}{2}}{4}$ $\frac{2\frac{1}{2}}{3}$ $\frac{4\frac{4}{5}}{12}$

ADDITION OF FRACTIONS

42. Before adding or subtracting fractions, we must reduce them to similar fractions; *i.e.* to fractions having the same denominator. It is usually best to reduce them to their *least* common denominator.

1. Add $14\frac{2}{3}$, $21\frac{5}{6}$, $18\frac{3}{4}$, $56\frac{5}{8}$.

You may arrange your work in this way:

$$\begin{array}{r} 14\frac{2}{3} = 14\frac{8}{12} \\ 21\frac{5}{6} = 21\frac{10}{12} \\ 18\frac{3}{4} = 18\frac{9}{12} \\ 56\frac{5}{8} = 56\frac{7\frac{1}{2}}{12} \\ \hline 109\frac{29\frac{1}{2}}{12} = 111\frac{7}{8} \text{ Ans.} \end{array}$$

Or you may arrange your work in this way:

$$\begin{array}{r} 24 \\ 14\frac{2}{3} \overline{) 16} \\ 21\frac{5}{6} \overline{) 20} \\ 18\frac{3}{4} \overline{) 18} \\ 56\frac{5}{8} \overline{) 15} \\ \hline 109\frac{69}{24} = 111\frac{7}{8} \text{ Ans.} \end{array}$$

2. Add $56\frac{1}{4}$
 $18\frac{5}{9}$
 $48\frac{7}{12}$

3. Add $124\frac{7}{10}$
 $258\frac{5}{12}$
 $141\frac{4}{15}$

4. Add $278\frac{5}{8}$
 $160\frac{1}{8}$
 $243\frac{9}{16}$

5. Add $12\frac{3}{4}$
 $17\frac{3}{10}$
 $36\frac{5}{8}$

6. Add $13\frac{2}{5}$, $24\frac{3}{8}$, $51\frac{1}{6}$.

7. Add $98\frac{1}{6}$, $31\frac{1}{4}$, $25\frac{1}{3}$.

8. Add $16\frac{5}{8}$, $71\frac{1}{2}$, $33\frac{1}{3}$.

9. Add $296\frac{1}{4}$, $45\frac{11}{15}$, $61\frac{9}{24}$.

10. Add $45\frac{5}{7}$, $150\frac{1}{10}$, $35\frac{4}{5}$.

11. Add $53\frac{2}{9}$, $36\frac{4}{15}$, $21\frac{3}{10}$.

12. Add $25\frac{2}{3}$, $56\frac{3}{4}$, $28\frac{5}{9}$, $15\frac{7}{8}$, $36\frac{7}{12}$, $48\frac{5}{6}$.

SUBTRACTION OF FRACTIONS

43. 1. Subtract $9\frac{4}{5}$ from $16\frac{2}{3}$.

You may arrange your work according to either model:

$$\begin{array}{r} 16\frac{2}{3} = 16\frac{4}{6} \\ 9\frac{4}{5} = 9\frac{4\frac{2}{5}}{5} \\ \underline{6\frac{1}{3}} \quad \text{Ans.} \end{array}$$

$$\begin{array}{r} 15 \\ 16\frac{2}{3} \overline{) 10} \\ \underline{9\frac{4}{3} 12} \\ 6 \frac{1}{3} \quad \text{Ans.} \end{array}$$

Find remainders:

2. $48\frac{9}{10}$
 $\underline{24\frac{3}{4}}$

3. $294\frac{5}{6}$
 $\underline{150\frac{11}{20}}$

4. $45\frac{2}{3}$
 $\underline{28\frac{1}{2}}$

5. $122\frac{3}{4}$
 $\underline{67\frac{2}{3}}$

6. $118\frac{3}{4}$
 $\underline{67\frac{8}{9}}$

7. $62\frac{1}{2}$
 $\underline{45\frac{7}{8}}$

8. 400
 $\underline{184\frac{5}{6}}$

9. $210\frac{1}{10}$
 $\underline{45\frac{5}{8}}$

10. $743\frac{4}{5}$
 $\underline{279\frac{3}{4}}$

Perform the operations indicated:

11. $140\frac{1}{8} - 76\frac{5}{6}$

12. $128\frac{1}{4} - 59\frac{3}{8}$

13. $147\frac{7}{8} - 49\frac{3}{10}$

14. $286 - 1\frac{4}{5}$

15. $56\frac{2}{3} + 35\frac{3}{4} - 12\frac{5}{6}$

16. $7\frac{1}{2} - 3\frac{9}{10} + 2\frac{1}{8}$

17. $(5\frac{5}{8} + 2\frac{1}{3}) - (3\frac{1}{2} + 6\frac{3}{4})^*$

18. $(3\frac{1}{3} - 2\frac{1}{5}) + (6\frac{1}{4} - 3\frac{3}{8})$

19. $12\frac{1}{2} - 6\frac{2}{3} + 24\frac{7}{8} - 15\frac{3}{4}$

20. $(16\frac{2}{3} + 18\frac{3}{4}) - (25\frac{5}{6} + 7\frac{1}{8})$

* The operations within a parenthesis are to be performed first and the result treated as a single number.

ORAL EXERCISES WITH SHORT METHODS

44. 1. $\frac{3}{4} + \frac{2}{3} = ?$

$$\begin{array}{cc} 9 & 8 \\ 4 & 3 \end{array}$$

Think this: $\frac{3}{4} + \frac{2}{3} = \frac{17}{12}$.

Say this: 3 times 3 are 9;
4 times 2 are 8; $9 + 8 = 17$;
 $4 \times 3 = 12$. The answer is
 $\frac{17}{12}$ or $1\frac{5}{12}$.

2. $\frac{3}{4} - \frac{2}{3} = ?$

$$\begin{array}{cc} 9 & 8 \\ 4 & 3 \end{array}$$

Think this: $\frac{3}{4} - \frac{2}{3} = \frac{1}{12}$.

Say this: 3 times 3 are 9;
4 times 2 are 8; $9 - 8 = 1$;
 $4 \times 3 = 12$. The answer is
 $\frac{1}{12}$.

3. $\frac{5}{8} + \frac{2}{3} = ?$

4. $\frac{3}{4} - \frac{1}{5} = ?$

5. $\frac{5}{6} + \frac{3}{5} = ?$

6. $\frac{5}{9} - \frac{1}{4} = ?$

7. $\frac{1}{8} + \frac{7}{10} = ?$

8. $\frac{5}{7} - \frac{1}{2} = ?$

9. $\frac{1}{8} + \frac{1}{5} = ?$

10. $\frac{1}{5} - \frac{1}{8} = ?$

NOTE. When both fractions have 1 for a numerator, add the denominators to get the numerator of the sum and multiply the denominators for the denominator of the sum. In example 9, we say $8 + 5 = 13$; 8 times 5 = 40; $\frac{13}{40}$ is the answer.

Similarly to subtract one fractional unit from another, subtract the denominators to get the numerator of the difference and multiply the denominators to get the denominator of the difference. In example 10, we say $8 - 5 = 3$; $8 \times 5 = 40$; $\frac{3}{40}$ is the answer.

11. $\frac{1}{5} + \frac{1}{12} = ?$

12. $\frac{1}{4} - \frac{1}{5} = ?$

13. $\frac{1}{9} + \frac{1}{7} = ?$

14. $\frac{1}{9} - \frac{1}{10} = ?$

15. $\frac{1}{10} + \frac{1}{25} = ?$

16. $\frac{1}{15} - \frac{1}{20} = ?$

17. $\frac{1}{8} + \frac{1}{10} = ?$

18. $\frac{1}{8} - \frac{1}{10} = ?$

MULTIPLICATION OF FRACTIONS

45. RULE. Multiply the numerators together for a new numerator and the denominators for a new denominator. Cancel if possible.

EXERCISES

- 46.** 1. Find $\frac{1}{6}$ of 80; $\frac{7}{25}$ of 125; $\frac{11}{12}$ of 240.
 2. How much is 18 times $1\frac{5}{9}$? 24 times $4\frac{5}{8}$?
 3. Solve: $\frac{2}{3}$ of $\frac{9}{10}$ of 25 = ?
 4. Multiply $12\frac{1}{2}$ by $6\frac{2}{3}$; $18\frac{3}{4}$ by $4\frac{1}{9}$.
 5. What is the product of $\frac{3\frac{3}{4}}{12}$ and $\frac{8}{9}$ of $3\frac{3}{4}$?

Multiply, using short methods:

6. $\frac{2}{7}$ by $1\frac{1}{5}$ 7. $15\frac{3}{4}$ by 20 8. $1\frac{1}{5}$ by $\frac{3}{7}$
 9. $8\frac{1}{3}$ by $\frac{1}{5}$ 10. $16\frac{2}{3}$ by $3\frac{3}{8}$ 11. $24\frac{3}{4}$ by 36
 12. $5\frac{5}{8}$ by $4\frac{1}{5}$ 13. $\frac{2}{5}$ by $\frac{3}{7}$ 14. $\frac{5}{12}$ by $\frac{9}{10}$

Simplify, using cancelation:

15. $\frac{5}{6}$ of $\frac{1}{2}$ of $\frac{3}{8}$ of $\frac{2}{5}$ 16. $\frac{2}{3} \times \frac{5}{8} \times \frac{2}{25}$ of 100
 17. $\frac{8}{15}$ of $\frac{2}{3} \times \frac{5}{2}$ of $3\frac{1}{8}$ 18. $\frac{3}{4}$ of $\frac{2\frac{1}{2}}{3} + \frac{3}{8}$ of $\frac{1}{6}$
 19. $2\frac{1}{2} \times 3 + 3\frac{1}{3} \times 5$ 20. $3\frac{3}{4} \times 6 - 2\frac{1}{5} \times 3$
 21. $\frac{5}{9} \times \frac{1}{2}$ of $18\frac{3}{4} + \frac{3}{8}$ of $16\frac{2}{3} \div \frac{1}{2}$
 22. $\frac{1}{15}$ of $5\frac{5}{8} \div \frac{7}{12}$ of $4\frac{1}{5}$ 23. $\frac{4}{15}$ of $180 - 3\frac{2}{3} \times 4\frac{1}{5}$

DIVISION OF FRACTIONS

47. RULE. Multiply the dividend by the reciprocal of the divisor. (Or invert the divisor and multiply.)

NOTE. In applying this rule, a whole number is regarded as the numerator of the fraction with denominator 1; and mixed numbers are reduced to improper fractions. The most frequent cause of error in division of fractions is the failure to distinguish which number is the divisor. Of course, if the dividend is inverted, the answer will be wrong.

SIGHT EXERCISES

48. Divide:

- | | | |
|--------------------------------------|-------------------------------------|---------------------------------------|
| 1. $\frac{5}{8}$ by $\frac{1}{4}$ | 2. $\frac{4}{5}$ by $\frac{2}{3}$ | 3. $\frac{9}{10}$ by $\frac{2}{3}$ |
| 4. $\frac{24}{25}$ by $\frac{8}{15}$ | 5. $\frac{15}{16}$ by $\frac{5}{8}$ | 6. $\frac{15}{24}$ by $\frac{25}{72}$ |
| 7. 10 by $2\frac{2}{3}$ | 8. 20 by $3\frac{1}{4}$ | 9. 35 by $4\frac{1}{5}$ |
| 10. $\frac{8}{9}$ by 5 | 11. $\frac{9}{20}$ by 36 | 12. $\frac{14}{15}$ by 8. |

How much is:

- | | | |
|--|---|---------------------------------------|
| 13. $3\frac{3}{5} \div 9?$ | 14. $4\frac{4}{5} \div 15?$ | 15. $5\frac{2}{5} \div 18?$ |
| 16. $6\frac{2}{3} \div 12\frac{1}{2}?$ | 17. $5\frac{2}{5} \div 12\frac{1}{10}?$ | 18. $50 \div 8\frac{1}{3}?$ |
| 19. $\frac{25}{4} \div \frac{3}{4}?$ | 20. $6\frac{3}{10} \div \frac{9}{10}?$ | 21. $9\frac{3}{8} \div 1\frac{7}{8}?$ |

WRITTEN EXERCISES

- 49.** 1. How many times is $8\frac{3}{4}$ contained in 100?
 2. How many times can $4\frac{5}{7}$ be subtracted from 60?
 3. What is the quotient of $120 \div 3\frac{3}{4}$?

4. What number must $3\frac{1}{8}$ be multiplied by to produce $18\frac{3}{4}$?

5. If the dividend was $66\frac{2}{3}$ and the quotient $2\frac{2}{5}$, what was the divisor?

Find quotients:

6. $5\frac{2}{15} \div \frac{7}{30}$ 7. $25\frac{2}{3} \div 44\frac{2}{5}$ 8. $17\frac{3}{5} \div 36\frac{3}{5}$

9. $250 \div 8\frac{1}{3}$ 10. $25\frac{5}{9} \div 2\frac{7}{9}$ 11. $400 \div 4\frac{3}{8}$

12. $116\frac{2}{3} \div 10$ 13. $341\frac{5}{7} \div 16$ 14. $1248\frac{3}{5} \div 24$

15. $27\frac{4}{5} \div 78\frac{2}{3}$ 16. $2741\frac{1}{9} \div 18$ 17. $172\frac{4}{5} \div 428\frac{1}{2}$

TYPE PROBLEMS INVOLVING FRACTIONAL RELATIONS OF NUMBERS

First Type Problem. To find a part of a number.

50. A man earns \$18 a week. What does he earn in $\frac{2}{3}$ of a week?

Explanation. The amount earned in $\frac{2}{3}$ of a week is $\frac{2}{3}$ of the amount earned in a week. Therefore, the amount is $\frac{2}{3}$ of \$18 = \$12. *Ans.*

Second Type Problem. To find what fraction one number is of another number.

A farmer had 60 sheep. He sold 15 sheep. What part of his flock did he sell?

Explanation. The part sold is 15 sheep out of a flock of 60 sheep, or $\frac{15}{60}$ of the flock. Therefore, the part sold is $\frac{15}{60} = \frac{1}{4}$. *Ans.*

Third Type Problem. Finding the whole when a fractional part of it is given.

First Case

(A) To find a number when a fractional part of it is given.

$\frac{3}{5}$ of Mary's age is 12 yr. How old is Mary?

Explanation. $\frac{3}{5}$ of Mary's age is 12 yr. Therefore, $\frac{1}{5}$ of her age equals $\frac{1}{3}$ of 12. Her full age equals

$$5 \text{ times } \left(\frac{1}{3} \text{ of } 12 \right) \text{ or } \frac{5}{3} \text{ of } 12 = 20 \text{ yr. } \textit{Ans.}$$

A shorter explanation is: $\frac{3}{5}$ of her age equals 12; then her full age equals $\frac{5}{3}$ of 12 = 20 yr. *Ans.*

Second Case

(B) To find a number when the number minus a fractional part is given.

A boy loses $\frac{1}{3}$ of his marbles. He then has 30 marbles. How many marbles had he at first?

Explanation. Since he lost $\frac{1}{3}$ of his marbles, 30 marbles is $\frac{2}{3}$ of the original number he had.

$\frac{2}{3}$ of the original number = 30 marbles.

$\frac{1}{3}$ of the original number = $\frac{1}{2}$ of 30 = 15 marbles.

$\frac{3}{3}$ of the original number = $3 \times 15 = 45$ marbles.

Or, a shorter explanation is:

$\frac{2}{3}$ of the number of marbles = 30.

All the marbles = $30 \div \frac{2}{3}$, or $\frac{3}{2}$ of 30 = 45. *Ans.*

Third Case

(C) To find a number, when the number plus a fractional part is given.

A boy added to his savings $\frac{2}{5}$ of the original amount. He then had \$21 in his bank. What was the original amount of his savings?

Explanation. Since he added $\frac{2}{5}$ to the original $\frac{5}{5}$, his savings are $\frac{7}{5}$ of the original amount.

$\frac{7}{5}$ of the original amount = \$21.

$\frac{1}{5}$ of the original amount = ($\frac{1}{7}$ of \$21) = \$3.

$\frac{5}{5}$ or the original amount = $5 \times 3 = \$15$. *Ans.*

Or, a shorter explanation is:

$\frac{7}{5}$ of the original amount = \$21.

The original amount = \$21 $\div \frac{7}{5}$ or $\frac{5}{7}$ of \$21 = \$15. *Ans.*

MISCELLANEOUS PROBLEMS IN FRACTIONS

ORAL PROBLEMS

51. 1. If $\frac{3}{5}$ of the amount of flour in a barrel is $16\frac{1}{2}$ lb., how many pounds are there in the barrel?

2. At \$ $2\frac{1}{4}$ a day, how much will a man earn in $7\frac{1}{2}$ da.?

3. A tub of butter contains 53 lb. From it the following quantities were sold: $3\frac{1}{4}$ lb., $7\frac{1}{2}$ lb., $6\frac{1}{4}$ lb., $\frac{1}{2}$ lb., $9\frac{3}{4}$ lb. How many pounds were sold? How many pounds remained in the tub?

4. How much do 9 lb. of butter cost at \$ $\frac{3}{5}$ a pound? How much change should I receive from \$10?

5. A hook $6\frac{1}{2}$ in. long is driven $4\frac{3}{4}$ in. into a wall. How much of the hook projects from the wall?

6. Find the cost of $6\frac{3}{4}$ lb. of fish at 12 ct. a pound.
7. Find the cost of $8\frac{1}{4}$ doz. eggs at 36 ct. a dozen.
8. $\frac{1}{4}$ lb. of meat costs $7\frac{1}{2}$ ct. Find the cost of $\frac{3}{4}$ lb.
9. $\frac{1}{4}$ lb. of meat costs 8 ct. How much meat may be bought for 64 ct.?
10. $\frac{1}{2}$ yd. of ribbon costs 14 ct. Find the cost of $3\frac{1}{2}$ yd.
11. $\frac{1}{2}$ yd. of ribbon costs 14 ct. How much ribbon may be bought for 70 ct.?
12. A boy earns $\$ \frac{3}{5}$ a day. In how many days will he earn $\$ 1\frac{4}{5}$? $\$ 3\frac{3}{5}$?
13. 6 yd. of cloth cost \$14. How much do 24 yd. cost?
14. 3 lb. of tea cost $\$ 1\frac{3}{4}$. Find the cost of 9 lb.
15. I spend 48 ct. for candies at 2 for 1 ct. How many candies should I receive for the same money at the rate of 3 for 1 ct.?
16. 48 T. of coal were divided among a number of families. Each family received $\frac{3}{4}$ T. How many families were supplied?

WRITTEN PROBLEMS

(Use paper and pencil only when the numbers are too large to be carried in the mind.)

52. 1. 12 bbl. of flour weighing 196 lb. each were bought at the rate of $4\frac{3}{4}$ ct. a pound and sold at the rate of $6\frac{1}{2}$ ct. a pound. Find the profit.

2. An express company delivered 96 packages, each weighing $7\frac{3}{4}$ lb. It charged $\frac{3}{4}$ ct. per pound. What was the total amount charged?

3. From a piece of cloth containing $68\frac{1}{4}$ yd., the following pieces were cut: $7\frac{3}{8}$ yd., $3\frac{3}{4}$ yd., $8\frac{1}{2}$ yd., 14 yd., $6\frac{1}{4}$ yd. What is the value of the remaining piece if sold at $\$1\frac{3}{4}$ a yd.?

4. A train that has been going at the rate of $18\frac{3}{4}$ mi. an hour increases its speed by $\frac{3}{8}$ of its former rate. At what rate per hour does it travel after the increase?

5. A train has a distance of 162 mi. to go. It travels at the rate of 36 mi. an hour for $2\frac{3}{4}$ hr. Then it decreases its speed by $\frac{1}{4}$ of its former rate. How long will it take the train to go the rest of the journey?

6. A school contains 3024 pupils. 5 out of every 9 were born in New York. How many were born in New York?

7. An agent agrees to take all the apples from Mr. Brown's orchard, at the rate of $\$2\frac{2}{5}$ per barrel. Mr. Brown sends the agent 475 bbl.

(a) If the agent pays $\$\frac{3}{10}$ per barrel for freight, how much do the apples cost him?

(b) For how much per barrel must the agent sell the apples to gain $\$1\frac{1}{4}$ on each barrel?

8. The cargo of a boat consisted of: 56 boxes, weighing $27\frac{1}{2}$ lb. each; 38 bales, weighing $108\frac{3}{4}$ lb.

each; 75 bbl., weighing $124\frac{1}{4}$ lb. each. Find the total weight of the cargo.

9. A boy is employed to address envelopes. He receives $\$ \frac{4}{5}$ for every 1000 envelopes he addresses. How much will he receive for addressing 13,750 envelopes?

10. A dealer bought 876 bbl. of apples at $\$ 3\frac{3}{4}$ a barrel. He sold $\frac{1}{2}$ of them at $\$ \frac{1}{4}$ a barrel, $\frac{1}{4}$ of them at $\$ 5\frac{1}{2}$ a barrel, and the remainder at $\$ 4\frac{3}{4}$ a barrel. Find the dealer's profit if his shipping expenses were $\$ 375$.

11. A grocer puts sugar up in packages, each package containing $3\frac{1}{2}$ lb. If the grocer makes a profit of $3\frac{1}{4}$ ct. on each package, how much profit will he make on 966 lb. of sugar?

12. If $3\frac{3}{4}$ yd. of cloth cost $\$ 4\frac{1}{2}$, what is the cost of $18\frac{1}{2}$ yd.?

13. $\frac{5}{8}$ of the distance between two cities is $170\frac{3}{8}$ mi. Find the entire distance.

14. $\frac{4}{9}$ of the pupils in a school are boys. If there are 585 girls, how many pupils are in the school? How many boys?

15. How many barrels of cement may be purchased for $\$ 57\frac{1}{2}$, if a barrel of cement costs $\$ 2\frac{1}{8}$?

16. How many pieces of rope each $1\frac{3}{4}$ ft. long may be cut from a piece 84 ft. long?

17. A vessel sailed $\frac{1}{4}$ of the distance between 2 ports the first day; $\frac{1}{5}$ of the distance the second

day; $\frac{1}{2}$ of the remainder the third day. There were 99 mi. left. Find the total distance.

18. A steel wire is cut into 12 equal pieces. Then one of the pieces is cut into 3 equal parts. If the length of one of the smaller pieces is $\frac{3}{8}$ of a foot, what was the length of the original wire?

19. A and B are 58 mi. apart, and are walking toward each other. A walks at the rate of $3\frac{1}{2}$ mi. an hour: B at the rate of $3\frac{3}{4}$ mi. an hour. How long will it take them to meet each other?

20. (a) A can do a piece of work in 9 hr. What part of it can he do in 1 hr.?

(b) B can do a piece of work in $7\frac{1}{2}$ hr. What part of it can he do in 1 hr.?

21. (a) A pipe fills a tank in 8 hr. What part of the tank will be filled in 1 hr.?

(b) A pipe empties a tank in 10 hr. What part of a tank will be emptied in 1 hr.?

22. A tank has two pipes. One will fill the tank in 4 hr., the other will empty it in 6 hr. How long will it take to fill the tank if both pipes are opened at the same time?

23. The distance between A and B is $3\frac{1}{3}$ times the distance between C and D. If $\frac{3}{5}$ of the distance between C and D is $38\frac{1}{4}$ mi., what is the distance between A and B? (Draw a diagram.)

24. The electric light used in a factory costs $\$ \frac{3}{5}$ every $4\frac{1}{2}$ hr. How much does it cost every 6 da. if light is used for $2\frac{3}{4}$ hr. a day?

25. The population of a city 3 yr. ago was 560,000. Each year during the last 3 yr. the population has increased $\frac{1}{20}$ of what it was the preceding year. Find the population.

26. If an aeroplane flies 56 mi. in $2\frac{2}{3}$ hr., in how many hours will it fly 96 mi.?

27. A merchant has two qualities of cloth. The better quality costs $3\frac{1}{3}$ times as much as the poorer quality. If $\frac{3}{4}$ yd. of the poorer quality costs \$ $1\frac{1}{2}$, find the cost of $7\frac{1}{2}$ yd. of the better quality.

28. A contractor completed $\frac{2}{5}$ of a job in $12\frac{1}{2}$ da. How much longer should it take to finish the job?

29. Two trains leave St. Louis at the same time. One goes in a westerly direction at the rate of $23\frac{7}{8}$ mi. an hour, the other goes in an easterly direction at the rate of $25\frac{2}{3}$ mi. an hour. How far apart will the trains be after traveling $6\frac{3}{4}$ hr.?

30. The register of a school is 480 pupils. All except 80 pupils were promoted. What fractional part was promoted?

31. A train runs 45 mi. the first hour, and $1\frac{1}{2}$ times as much the second hour; the third hour its speed is $\frac{3}{4}$ of that of the second hour. Find the number of miles the train travels in the three hours.

32. A man invests his money in railroad bonds and in real estate. His income from the latter is

\$ 360, or $\frac{3}{5}$ of his entire income. Find the income from bonds.

33. A man's pay was increased by $\frac{1}{20}$, after which he received \$ 4.20 a day. What did he receive per day before the increase?

34. The Twentieth Century Express runs at an average speed of 45 mi. an hour. This is $2\frac{7}{8}$ times the speed of Stephenson's first locomotive of 1829. Find the rate of the first locomotive.

35. On a map, a line $2\frac{3}{4}$ in. long represents a distance of 132 mi. What distance is represented by a line $8\frac{1}{2}$ in. long?

36. A piece of glass $6\frac{3}{4}$ in. by $6\frac{1}{2}$ in. is cut from a pane $22\frac{1}{2}$ in. by $19\frac{1}{2}$ in. What fractional part of the whole pane is left? (Draw a diagram.)

DENOMINATE NUMBERS

WRITTEN EXERCISES

53. 1. How many feet in $\frac{5}{8}$ of a mile?
2. Reduce $\frac{3}{4}$ sq. ft. to inches.
3. How many feet in 32.18 chains?
4. What is the length in feet of 21.8 fathoms?
5. How many cubic feet should 12 cords of wood measure?
6. What part of an ounce is $3\frac{1}{3}$ pwt. of gold?
7. Two thirds of an hour is what part of a day?
8. Reduce 275 lb. to the decimal of a ton.

9. What decimal part of a gallon is $.66\frac{2}{3}$ of a pint?
10. How many grains in $.25$ of a pound avoirdupois?
11. Express 72 lb. of wheat in terms of bushels.
12. What fractional part of a quadrant is $18^{\circ} 43'$?
13. Express 18s. 5d. as the decimal fraction of £1.
14. How many grains of gold in a nugget weighing 1 oz. 7 dwt.?
15. What is the weight of $\frac{2}{5}$ of a barrel of flour?

Compound Numbers

Addition and Subtraction

EXERCISES

54. Find sums :

1.	2 cwt.	10 lb.	2.	10 gal.	3 qt.	
	1	50		4	1	1 pt.
	18	78		12	2	1
	75	80		3	0	1
	12	3		16	3	

3.	5 yd.	2 ft.	10 in.	4.	1 yr.	8 mo.	12 da.
	3	1	4		3	5	18
	1	1	8		1	2	28
	6	0	5			7	15
	12	2	7		3		6
	<hr/>				<hr/>		

5.	£ 3	8s.	2d.	6.	1 lb.	3 oz.	18 pwt.
	4	10	6		2	10	16
	1	9	10		7		12
	5		3			11	9
	6	7	9				

Find remainders :

7.	10 bu.	3 pk.	5 qt.	8.	92°	15'	20''
	8	2	7		75	45	45

9.	yr.	mo.	da.	10.	yr.	mo.	da.
	1913	4	15		1913	6	25
	1910	8	20		1911	11	12

11.	£ 75	6s.	3d.	12.	1 lb.	6 oz.
	.55	10	10			11 17 pwt.

Multiplication and Division

PROBLEMS

55. 1. How much wine will 10 casks hold, each having a capacity of 18 gal. 3 qt. 1 pt. ?

2. A ship sailing along the 40th parallel of latitude traveled $68^{\circ} 6'$ in 12 da. How far did it sail on an average each day ?

3. A train moves 80 ft. per second. What is its rate expressed in miles per hour ?

4. A wheat barge had 60 bins with a capacity of 150 bu. 3 pk. each. What is the total capacity expressed in bushels? What is the capacity expressed in pounds?

5. An invoice for 250 yd. of carpet costs £412 10s. What is the price per yard in English money? In U. S. money?

6. In covering a half mile, the wheels of my bicycle revolved 510 times. Find the circumference of the wheels.

7. What is the cost of 200 yd. of Scotch Tweed at 3s. $4\frac{1}{2}$ d. per yard?

8. A coal wagon carries when filled 3 T. 2 cwt. 50 lb. of coal. How much will it convey in 20 trips, if filled on each trip?

9. How many square feet in a plot, if there are 18 plots in an acre?

10. Divide 280 bu. 3 qt. by 35.

11. Find the weight of a dozen silver plates, allowing 7 oz. 8 dwt. for each plate.

12. A goldsmith used 4 oz. 7 pwt. 12 gr. of gold in making 25 bracelets. How much gold did each bracelet contain?

13. A tile layer lays 150 tiles in a day, each tile having a surface of 16 sq. in. How many square feet of surface can he cover in 18 da.?

14. How many bottles containing $\frac{7}{8}$ of a quart can be filled from a barrel of vinegar containing $31\frac{1}{2}$ gal?

15. A canal barge contains 320 T. of coal. How many bucket loads of 150 lb. each will it take to remove the cargo?

Foreign Moneys

English Money

56. English money is the currency used in Great Britain and Ireland. The unit is the pound sterling (£), a gold coin, worth \$4.8665, or about \$5.00, in U. S. money. The shilling is worth about 25 cents, and the penny is worth about 2 cents of our money.

Table of English Money

4 farthings (far.)	= 1 penny (d.)
12 pence	= 1 shilling (s.)
20 shillings	= 1 pound sterling (£)
21 shillings	= 1 guinea (G.)

£ is written **before** the number, *e.g.* £10.

The currency systems of several other countries of commercial importance are shown in the following table:

Country	Standard	Divisions	U. S. Value
Germany	1 Mark (M.)	= 100 Pfennig (pf.)	= \$.238
France	1 Franc (fr.)	= 100 Centimes (c.)	= .193
Italy	1 Lira	= 100 Centesimi	= .193
Austria	1 Crown	= 100 Heller	= .203
Russia	1 Ruble	= 100 Kopecks	= .515
Mexico	1 Peso		= .498
Brazil	1 Milreis		= .546
Netherlands	1 Florin or Guilder		= .402

The approximate value in U.S. money of a mark is about 25 cents; of a franc, a lira, and an Austrian crown, about 20 cents; of a ruble, a peso, and a milreis, about 50 cents.

EXERCISES

57. Find the approximate value in U. S. money of :

- | | | |
|---------------------|-----------------|------------------|
| 1. £ $2\frac{1}{2}$ | 2. £ 10 10s. | 3. 9s. 6d. |
| 4. 60 M. | 5. 250 fr. | 6. 5 M. 50 pf. |
| 7. 50 fr. | 8. 400 M. | 9. 10 fr. 25 c. |
| 10. 50 pf. | 11. 20s. | 12. £ 300 |
| 13. 50 c. | 14. 40 pf. | 15. 150 M. |
| 16. 50s. | 17. 40 c. | 18. 150 fr. |
| 19. 100 lira | 20. 60 crowns | 21. 800 rubles |
| 22. 200 pesos | 23. 30 milreis | 24. 120 florins |
| 25. 35 lira | 26. 500 crowns | 27. 380 rubles |
| 28. 99 pesos | 29. 600 milreis | 30. 900 guilders |

To change Foreign Money to U. S. Money

58. 1. Find the exact value in U. S. money of 15 fr. 35 c.

Statement

1 fr. = \$.193

15.35 fr. = ?

Analysis

15.35 francs are worth
15.35 times the value of 1
franc.

$15.35 \times \$.193 = \$2.96.$

2. Change 19 M. 75 pf. to U. S. money.

1 M. = 23s. $\therefore 19.75 \text{ M.} = 19.75 \times \$.238 = \$4.70.$

3. How much is £5 6s. worth in U. S. currency?
 $\text{£ } 5 \text{ 6s.} = \text{£ } 5\frac{6}{20} = \text{£ } 5.3.$

$$\text{£ } 1 = \$4.8665. \quad \therefore \text{£ } 5.3 = 5.3 \times \$4.8665^* = \$25.79.$$

4. Change £78 16s. 9d. to U. S. money.

$$\begin{array}{r} 12 \overline{) 9.00} \\ 20 \overline{) 16.75} \\ \hline \text{£ } 78.8375 \end{array}$$

$$\begin{aligned} 9d. &= .75s. \\ 16.75s. &= \text{£ } .8375 \\ \text{£ } 78.8375 &= 78.8375 \times \$4.8665^* \\ &= \$383.66 \end{aligned}$$

RULE. Change the lower denominations to a fractional part of the standard unit; then change to U. S. money.

EXERCISES

59. Give the exact equivalent of the following in U. S. money:

- | | | |
|-----------------|------------|-----------------|
| 1. 26 M. 20 pf. | 5. £4 15s. | 9. £3 12s. 2d. |
| 2. 18 fr. 15 c. | 6. £6 10s. | 10. 29 M. 5 pf. |
| 3. 27 M. 12 pf. | 7. £3 5s. | 11. 16 M. 2 pf. |
| 4. 30 fr. 24 c. | 8. £5 14s. | 12. 36 fr. 6 c. |

EXERCISES

To change United States Money to Foreign Money

60. 1. Change \$50.75 to French money.

Statement
 $\$.193 = 1 \text{ fr.}$
 $\$50.75 = ?$

Analysis
 The number of francs will equal
 the number of times that \$.193 is
 contained in \$50.75.

$$50.75 \div .193 = 262.95, \text{ or } 262 \text{ fr. } 95 \text{ c.}$$

* NOTE. $\$.487 = \text{£ } 1$ is sufficiently accurate for most purposes.

2. Change \$25 to English money.

Statement
 $\$4.87 = \text{£ } 1$
 $\$25 = ?$

Process
 $25 \div 4.87 = 5.13$ or $\text{£ } 5.13$
 $\text{£ } .13 = 2.6\text{s.} ; .6\text{s.} = 7.2\text{d.}$
 $\text{£ } 5 \text{ } 2\text{s. } 7\text{d.}$ *Ans.*

EXERCISES**61. Change the following to exact equivalents:**

	FRENCH MONEY	GERMAN MONEY	ENGLISH MONEY
1. \$ 10	?	?	?
2. \$ 25	?	?	?
3. \$ 100	?	?	?
4. \$ 250	?	?	?
5. \$ 350	?	?	?
6. \$ 225	?	?	?
7. \$ 2000	?	?	?

PROBLEMS

(Use approximate values.)

62. 1. A boy who was learning the trade of bricklaying in England received 6s. per week. How much is that in U. S. money?

2. Four years later the boy received an increase of 8s. Find his wages in U. S. money.

3. An English carpenter receives $10\frac{1}{2}\text{d.}$ per hour. He works 50 hr. per week. Find his wages in U. S. money.

4. An overcoat was bought in London for £ 2 3s. How much is that in U. S. money?

5. The price of a fan in Paris is 20 fr., the price in New York is \$ 6. How much more do I pay for the fan in New York? (Give answer in U. S. money.)

6. I paid 22 M. for a pair of shoes in Berlin. The same kind of shoes cost \$ 4 in New York. How much more did the shoes cost in New York? (Give answer in U. S. money.)

7. My bill at a German hotel was 6 M. 50 pf. per day for 6 da. State, in U. S. money, the amount paid in 6 da.

8. The cost of a railroad ticket was 10 M. 20 pf. State the equivalent amount in U. S. money.

9. A trip on a boat in France cost 3 fr. 25 c. Find the equivalent amount in U. S. money.

The wages of various trades in New York are given below. Find the *approximate equivalent* of the *weekly* wage in English money.

10. Bricklayer, 70 ct. an hour, 44 hr. a week.

11. Carpenter, $62\frac{1}{2}$ ct. an hour, 44 hr. a week.

12. Laborer, $37\frac{1}{2}$ ct. an hour, 44 hr. a week.

13. Painter, \$ 4 a day, $5\frac{1}{2}$ da. a week.

14. Plumber, \$ 5 a day, $5\frac{1}{2}$ da. a week.

15. Railroad engineer, \$ 4.44 a day.

16. Railroad machinist, \$ 3.25 a day.

The wages of various trades in London are stated below. Find the equivalent of the *weekly* wage in U. S. money.

17. Bricklayer, $10\frac{1}{2}d.$ per hour, 50 hr. a week.
18. Mason, $11\frac{1}{2}d.$ per hour, 50 hr. a week.
19. Painter, $9d.$ per hour, 50 hr. a week.
20. Plumber, $11d.$ per hour, 50 hr. a week.
21. Tailor, $7d.$ per hour, 55 hr. a week.

PROBLEMS

(Use exact values.)

63. State the cost of the following articles (*a*) in French money, (*b*) in German money:

1. A table worth \$ 60.
2. A piano worth \$ 500.
3. An automobile worth \$ 1800.
4. A set of books containing 12 vols. at \$ 2.50 per volume.
5. 80 yd. of cloth at \$ 2.25 per yard.
6. Find the value of the table (number 1) in English money.
7. Find the value of the piano (number 2) in English money.
8. Find the value of the automobile (number 3) in English money.
9. Find the value of the 80 yd. of cloth (number 5) in English money.

BUSINESS FORMS

FOR READING AND DISCUSSION

64. Goods for family use or consumption are usually bought from retail merchants. The form of the **bill** commonly sent by retail dry goods merchants to their customers is as follows:

Bills rendered monthly. All claims for errors must be made within ten days after receipt of goods.

NEW YORK, January 2, 1913.

MRS. R. T. FULLER,
147 Madison Ave.

Bought of ROBBINS & SMITH,

IMPORTERS AND RETAILERS OF DRY GOODS,
West 23d Street.

Salesperson Date

	Dec.	<i>Dr.</i>					
214	6	1 Coat		32	00		
		7½ yd. Suiting	1.25	9	38		
		2 yd. Elastic	.11		22	41	60
336	7	6 yd. Serge	.95	5	70		
		2 pc. Binding	.23		46	6	16
38	9	2 Suits	8.90	17	80	17	80
781	15	¼ yd. Veiling	.60		15		
		2 pr. Gloves	1.50	3	00	3	15
459	19	1 bx. Hdkfs.		2	50	2	50
						71	21
		<i>Cr.</i>					
	22	1 bx. Hdkfs.				2	50
						68	71

Verify each of the items on this bill.

WRITTEN EXERCISES

1. Prepare a bill for goods sold by Robbins and Smith to you as follows: May 2, 1913, $12\frac{1}{2}$ yd. Cloth @ \$ 1.85, # 28*; 1 pr. Gloves \$ 2.80, # 16; 1 Jabot \$.85. May 7, 1913, 15 yd. Ribbon @ \$.48, # 47; 6 Hdkfs. @ \$.35. May 13, 1913, 23 yd. Sheeting @ \$.49, # 154; 6 Pillow Cases @ \$.75. May 20, 1913, $8\frac{1}{2}$ yd. Silk @ \$ 2.25, # 890, 4 yd. Tulle @ \$.34, # 273.

2. Prepare a bill for the following goods sold to you by E. M. McLoughlin: June 1, 19—, 1 bbl. Pillsbury Flour \$ 5.75; 25 lb. Sugar @ \$.04 $\frac{1}{4}$; 30 lb. Raisins @ \$.09; 1 gal. Porto Rico Molasses \$.60; 1 gal. Olive Oil, \$ 2.75; 2 doz. cans Tomatoes @ \$ 1.38; 8 jars Jam @ \$.22; 15 bot. C. & B. Gherkins @ \$.28; 3 gross Matches @ \$.62; 18 lb. Rice @ \$.07 $\frac{1}{2}$.

3. Make out in proper form a bill of goods sold by a local dealer, find the footing, and receipt the bill.

FOR READING AND DISCUSSION

65. Retail merchants do not usually manufacture the goods they sell nor do they purchase their merchandise directly from the manufacturer. They buy their goods from commission merchants or from jobbing merchants.

* This number identifies the salesperson. The symbol # stands for number when placed *before* a numeral.

The following letter is an **order** for goods from The Empire Hardware Company, retailers, to the H. C. Ackerson Company, jobbing merchants.

NEWBURG, N.Y., April, 11, 1913.

THE H. C. ACKERSON CO.

345 West Broadway, New York.

GENTLEMEN :

Please ship us at once by fast freight :

$\frac{1}{2}$ doz. Ladders, # 87
3 doz. Ladder Chairs, # 593
15 doz. Glass Lamps, # 623
 $7\frac{1}{4}$ doz. Washbowls, # 183
5 gro. Glasses, # 96

Yours truly,

EMPIRE HARDWARE CO.

By F. T. LEWIS.

66. Write the letter of The H. C. Ackerson Co., acknowledging the receipt of the order.

67. Observe these points in writing letters ordering goods :

- 1.** The items are arranged one on a line.
- 2.** The quantity and the catalogue number of each article are definitely stated.
- 3.** Shipping directions are given.

68. Upon shipping the goods ordered, The H. C. Ackerson Co. sent the following bill or **invoice** to The Empire Hardware Co.

All claims must be made within five days after receipt of goods.

NEW YORK, April 15, 1913.

THE EMPIRE HARDWARE CO.,
Newburg, N. Y.

Bought of THE H. C. ACKERSON CO.,

Terms, n/30.

HARDWARE, CROCKERY, AND GLASSWARE.

Via Freight.

345 West Broadway.

87	$\frac{1}{2}$ doz. Ladders	@ 9.00	4	50
593	3 doz. Ladder Chairs	@ 6.12 $\frac{1}{2}$	18	38
623	15 doz. Glass Lamps	@ 4.17	62	55
183	5 doz. Washbowls	@ 10.50	52	50
96	5 gro. Glasses	@ 9.50	47	50
			185	43

1. Compare this invoice with the order on page 56.

2. Upon receipt of the goods indicated in this invoice, The Empire Hardware Co. found that they were 3 doz. lamps short. Write a letter asking credit for the shortage.

NOTES. Term n/30 means that payment is to be made without discount in 30 da.

Terms 3/10, 2/30, 1/90, etc., mean that payment is to be made in 10, or 30, or 90 da. with a discount of 3 %, 2 %, or 1 %, respectively.

Terms 3/10, n/30 mean that 3 % will be allowed if payment is made in 10 da. or that the whole amount is due in 30 da.

69. A receipted invoice is shown below :

BOSTON, MASS., Oct. 14, 1913.

MR. EDWARD E. FOSTER,

123 Copley Square, City.

To ARMSTRONG BROTHERS, Dr.

Terms: 3/10, 1/30, net 60 da.

# 15	136 Mahogany Dining Chairs	12.50	1700			
# 314	120 Oak Dining Chairs	9.75	1170			
# 81	6 Oak Chiffoniers	15.25	91	50		
# 23 B	12 Brass Bedsteads	16.90	202	80		
# 294	5 Mahogany Parlor Tables	17.75	88	75		
# 709	28 Woven Wire Mattresses	3.65	102	20	3355	25
	Less 3 %				100	65
					3254	60

Received Payment, Oct. 24, 1913

ARMSTRONG BROS.

per R. T. A.

70. The bill was paid by a **check** on the Merchants Bank in which E. E. Foster keeps his account.

No. <i>~~~~~</i>	Boston, Mass., <i>~~~~~</i> Oct. 23, <i>~~~~~</i> 1913.
The Merchants Bank	
PAYABLE THROUGH THE CLEARING HOUSE	
Pay to <i>Armstrong Brothers</i> <i>~~~~~</i> or order	
<i>Thirty-two Hundred Fifty-four</i> $\frac{60}{100}$ <i>~~~~~</i> Dollars	
<i>\$ 3254</i> $\frac{60}{100}$	<i>~~~~~</i> Edward E. Foster. <i>~~~~~</i>

71. Acknowledgment of the payment of an invoice is generally made on a separate form of **receipt**, like the following:

BOSTON, MASS., Oct. 24, 1913.	
Received of Edward E. Foster ~~~~~	
Thirty-two Hundred Fifty-four $\frac{60}{100}$	~~~~~ dollars
in settlement of invoice of Oct. 14, 1913.	
ARMSTRONG BROTHERS.	
\$ 3254 $\frac{60}{100}$	per R. T. A.

EXERCISES

72. Write the letter ordering goods, and the letter acknowledging the order, the letter advising of shipment, the invoice, the check, and the receipt used in each of the following sets of transactions:

1. May 12, 19 —. Jordan and Martin, retailers of hardware, Pittsburgh, Pa., ordered of T. E. Fisk, Philadelphia, Pa., jobbing merchants, the following: 16 doz. carpenter's chisels, No. 16, @ \$ 3.85; 15 doz. hammers, No. 59, @ \$ 6.15; 43 doz. doorknobs, No. 77, @ 84 ct.; 8 grindstones, No. 9, @ \$ 4.12; 22 doz. saws, No. 68, @ \$ 19.50; 18 kegs wire nails 8 d, @ \$ 2.85; 35 sets knives and forks, No. 101, @ \$1.38; 6 Star ranges, No. 85, @ \$ 10.50.

The goods were shipped May 20, by fast freight via Pennsylvania R.R.; the terms of sale were 8/10, 3/30, n/60. The bill was paid May 30.

2. Aug. 14, 19 . R. M. White & Co., Mobile, Ala., ordered of the J. M. Galton Manufacturing Company, Grand Rapids, Mich., the following :

10 Cheval Glasses, # 29 A, @ \$ 23.56.

124 Cane-seat Dining Chairs, # 71, @ \$ 2.15.

18 Oak China Closets, # 35, @ \$ 27.90.

9 Mahogany China Closets, # 43, @ \$ 48.75.

16 Empire Tables, # 115, @ \$ 42.18.

4 Library Tables, # 193 C, @ \$ 22.35.

12 Oak Bookcases, # 12 B, @ \$ 14.28.

27 Turkish Rockers, # 51, @ \$ 14.89.

36 White Iron Bedsteads, # 72 B, @ \$ 4.75.

The goods were shipped Sept. 8, by freight via Michigan Southern R.R.; the terms of sale were 10/10, 5/30, n/90.

NOTE. For forms of notes and drafts, see chapters on Promissory Notes.

Accounts

FOR READING AND DISCUSSION

73. Every individual or firm that engages in business must keep a record of goods bought and sold, of money received and paid out, of what he owes to others and what others owe him, and of all other transactions that have a money value. The keeping of this record in a systematic way is the art of bookkeeping. The record is kept in a number of books. The most important book is the **ledger**, which consists of a number of **accounts**.

74. An **account** is a formal * record of the business transactions relating to the same person or thing.

75. A **personal account** records the transactions with the person named in the title.

76. Every account has two sides, the left-hand or **debit** side, and the right-hand or **credit** side.

77. The debit side of the personal account shows the items **against** the person, *i.e.* what he owes us or what we have paid to him.

78. The credit side of the personal account shows the items **in favor of** the person, *i.e.* what we owe him or what he has paid to us.

79. A personal account.

Dr. **EMPIRE HARDWARE CO.** *Cr.*
 NEWBURG, N.Y.

1913			*			1913			+		
Apr.	15	Mdse.	110	185	43	Apr.	24	Cash	56	100	00
	20	Mdse.	115	216	89		28	Cash	58	250	80
	25	Mdse.	121	57	92		29	Cash	59	65	35
	30	Mdse.	124	614	28	Apr.	31	Balance		658	37
				1074	52					1074	52
May	1	Balance		658	37						

This "personal account" records our dealings with the Empire Hardware Co. for April 1913 as follows :

April 15, the Empire Hardware Co. bought from us merchandise amounting to \$185.43; April 20, they bought a

* Formal here means according to regular and established forms and methods.

bill of \$ 216.89; April 24, they paid us \$ 100; April 25, they bought a bill of \$ 57.92; April 28, they paid us \$ 250.80; April 29, they paid \$ 65.35; April 30, they bought a bill of \$ 614.28. Their debits for the month amounted to \$ 1074.52 and their credits to \$ 416.15. They owe us the balance, \$ 658.37.

Observe that the "balance" is entered on the smaller side of the account, so that when both sides are added or "footed," they will be equal, *i.e.* the two sides *balance* as the scales balance when the sugar on one side weighs as much as the pound weight on the other side. This entry is usually made in red ink to show that it does not record a cash payment or other real transaction.

After the account is ruled or "closed" the balance is brought down to the other side. It is entered in black ink because it records the actual state of affairs, to wit: the Empire Hardware Co. owes us \$ 658.37.

The column marked * shows the pages of the "sales book" from which these items were taken; and the column marked † shows the pages of the "cash book" from which these items were taken.

EXERCISES

80. 1. Make the following entries in the account of The Keystone Manufacturing Company:

Jan. 1. They owe us a balance of \$ 215.85.

NOTE. Enter on Debit side as "Balance \$ 215.85."

Jan. 3. Received from them cash on account, \$ 150.

Jan. 4. Sold them merchandise on account, \$ 108.92.

Jan. 6. They returned for credit goods ordered by mistake in shipment of Jan. 4, \$ 28.94.

Jan. 7. Sold them merchandise on account, \$ 76.54.

Jan. 8. Received their note at 30 da. in settlement of balance unpaid on Jan. 3, \$ 65.85.

Jan. 9. Sold them merchandise on account, \$ 200.

Jan. 10. Received from them T. J. Smith's note in their favor which they have indorsed to us, \$ 50.

Jan. 20. We have drawn on them a draft at sight, in favor of E. F. Locke, for \$ 150.

Jan. 21. Sold them merchandise on account, \$ 318.17.

Jan. 24. Received from them their draft at sight on Marshall, Field & Co., Chicago, Ill., in our favor for \$ 115.85.

Jan. 30. We discount the balance of their account at $2\frac{1}{2}\%$ and they remit a check in settlement, which we accept.

Jan. 30. Sold them merchandise for \$ 892.67.

Jan. 31. Received from them a receipted freight bill for \$ 2.80 which should have been paid by us on shipment of Jan. 30.

What is the balance Jan. 31?

Write the notes mentioned on Jan. 8 and Jan. 10.

Write the check mentioned on Jan. 30.

Write a letter to the Keystone Manufacturing Company acknowledging the receipt of T. J. Smith's note.

2. Make the following entries in the account of Rogers and Marshall and find the balance July 31:

July 1. We owe them a balance of \$ 116.75.

July 2. Bought of them merchandise on account, \$ 62.72.

July 3. Paid them cash, \$ 100.

July 5. We returned to them for credit, goods purchased on July 2, amounting to \$ 9.75.

July 9. We gave them our note at 60 da. in settlement in full of account.

81. A **cash account** records on its debit side the money received and on its credit side the money paid out.

Dr.

CASH

Mo.	DAY	EXPLANATION	PAGE	\$	Ct.
19-					
Feb.	1	Balance		735	84
		J Duke & Co.		248	61
		G. Pall		75	94
	2	Sales		57	62
		F. Baldwin		24	73
		S. Reich		38	07
	6	Sales		87	56
		E. Short & Co.		15	45
				1287	82
				1287	82
Feb.	6	Balance		237	65

July 13. Bought of them merchandise on account, \$ 175.

July 15. We indorsed to them R. E. Swift's note at 30 da. in our favor for \$68.50.

July 23. Bought of them merchandise on account, \$ 95.70.

July 25. They allow us a claim for shortage in their shipment to us of Jan. 23, \$ 8.35.

July 30. They allow us a discount of 2 % on balance now due, and we remit to them a draft in their favor drawn on Hiram King & Co., Buffalo, N. Y., for \$ 85.61.

Study the cash account here given.

CASH

Cr.

Mo.	DAY	EXPLANATION	PAGE	\$	Ct.
19-					
Feb.	1	Expense		62	50
	2	S. Miller & Son		79	60
		Purchases		218	77
	3	G. Smith		25	00
		Personal		50	00
	5	Expense		3	75
		J. Weil & Co.		35	05
	6	M. Dyer		17	50
				1050	17
	6	Balance		237	65
				1287	82

NOTES. 1. The names of persons are those from whom money was received (Dr.) or to whom money was paid (Cr.). We have personal accounts with all the persons named.

2. "Sales" means "sales for cash" for which no one owes us any money.

3. "Expense" means rent, light, telephone, stationery, and, perhaps, wages and other items.

4. "Purchases" means "purchases for cash" for which we owe no one.

5. "Personal" means money taken by the proprietor for his personal use.

6. The "Balance" shows the cash on hand. This item must be verified by counting the actual cash and by having the bank book "balanced" at the bank.

1. Make the following entries in the Cash Account:

Sept. 1, 19—. We invested cash in the business, \$ 2500.

Sept. 1, 19—. Paid rent, \$ 50.

Sept. 1, 19—. Paid for furniture and fixtures, \$ 350.

Sept. 1, 19—. Paid for books and stationery, \$ 16.50.

Sept. 1, 19—. Received from E. R. Jones, \$ 250.

Sept. 2, 19—. Purchased merchandise for Cash, \$ 175.

Sept. 2, 19—. Cash sales, \$ 24.50.

Sept. 3, 19—. Paid freight bill, \$ 12.15.

Sept. 4, 19—. Received from E. O. Wilson, \$ 316.80.

- Sept. 5, 19—. Cash sales, \$ 31.87.
Sept. 5, 19—. Paid S. E. Field, \$118.09.
Sept. 8, 19—. Paid my note in favor of L. I. Boyle, \$ 75 with interest \$ 10.25.
Sept. 15, 19—. Cash sales, \$ 428.91.
Sept. 29, 19—. Cash sales, \$ 609.12.
Sept. 29, 19—. Received payment of E. M. Stafford's note, \$ 87.50.
Sept. 30, 19—. Paid salaries of clerks, \$ 54.75.
Sept. 30, 19—. Withdrew for personal use, \$ 35.
What should be the balance of cash on hand Sept. 30 ?

2. Enter the following transactions in a Cash Account :

- Jan. 1, 19—. Cash on hand, \$ 1287.54.
Jan. 1, 19—. Paid for stationery, \$ 21.16.
Jan. 1, 19—. Cash sales, \$ 187.28.
Jan. 2, 19—. Paid rent, \$ 125.
Jan. 2, 19—. Purchased horse and wagon, \$ 275.
Jan. 3, 19—. Paid office salaries, \$ 78.50.
Jan. 3, 19—. Withdrew for personal use, \$ 50.
Jan. 4, 19—. Cash sales, \$ 328.96.
Jan. 5, 19—. Paid R. S. MacAllister for his invoice of Dec. 28, \$ 894.35.
Jan. 5, 19—. Cash purchases, \$ 287.92.
Jan. 6, 19—. M. A. McCall paid on account, \$ 350.

Close the account and find the amount of cash on hand.

PERCENTAGE

MEANING OF PER CENT RELATIONS

FOR READING AND DISCUSSION

82. 1. A grocer has 250 apples. 20 per cent of them are red apples. How many red apples has he?

When we say that 20 per cent of the apples are red, we mean that 20 apples out of every hundred are red.

Therefore, 20 per cent means .20, or $\frac{20}{100}$, or $\frac{1}{5}$.

.20 or $\frac{1}{5}$ of 250 apples = 50 apples.

2. The distance from town A to town B is 300 miles. The distance between towns A and C is 75 per cent of the distance from A to B. How far is A from C?

The distance between A and C is 75 per cent or .75 of the distance from A to B.

75 per cent means .75, or $\frac{75}{100}$, or $\frac{3}{4}$.

$\frac{3}{4}$ of 300 miles = 225 miles.

83. Per cent, written %, means **hundredths**. It is an abbreviation of the Latin *per centum*.

20 per cent is written 20%; it means 20 out of every 100, or .20, or $\frac{1}{5}$.

$33\frac{1}{3}$ per cent is written $33\frac{1}{3}\%$; it means $33\frac{1}{3}$ out of every 100, or $.33\frac{1}{3}$, or $\frac{1}{3}$.

$\frac{1}{2}$ per cent is written $\frac{1}{2}\%$; it means $\frac{1}{200}$ out of every 100, or $.00\frac{1}{2}$, or $\frac{1}{200}$.

Changing Decimal Fractions and Common Fractions to Per Cents

EXERCISES

84. Change the following decimal fractions to per cents :

Process

$$.09 = 9\% \quad .035 = .03\frac{1}{2} = 3\frac{1}{2}\%$$

First read as hundredths; then state as per cent :

- | | | | |
|--------|----------------------|----------|-----------------------|
| 1. .07 | 6. .02 $\frac{1}{2}$ | 11. .003 | 16. 10.00 |
| 2. .15 | 7. .125 | 12. .110 | 17. 5.50 |
| 3. .29 | 8. .005 | 13. 1.5 | 18. .625 |
| 4. .18 | 9. .0025 | 14. 2.25 | 19. .0325 |
| 5. .06 | 10. .001 | 15. 3.00 | 20. .05 $\frac{1}{2}$ |

EXERCISES

85. Change the following common fractions to per cents :

1. $\frac{1}{2}$.

Process

$$\frac{1}{2} = \frac{50}{100} = 50\%$$

First change to hundredths; then state as per cent.

- | | | | |
|-------------------|--------------------|---------------------|--------------------|
| 2. $\frac{1}{4}$ | 8. $\frac{1}{5}$ | 14. $\frac{2}{7}$ | 20. $\frac{7}{10}$ |
| 3. $\frac{1}{3}$ | 9. $\frac{4}{5}$ | 15. $\frac{7}{12}$ | 21. $\frac{3}{20}$ |
| 4. $\frac{2}{3}$ | 10. $\frac{1}{8}$ | 16. $\frac{4}{15}$ | 22. $\frac{3}{5}$ |
| 5. $\frac{3}{4}$ | 11. $\frac{5}{8}$ | 17. $\frac{20}{25}$ | 23. $\frac{9}{10}$ |
| 6. $\frac{1}{6}$ | 12. $\frac{1}{10}$ | 18. $\frac{5}{7}$ | 24. $\frac{5}{6}$ |
| 7. $2\frac{1}{2}$ | 13. $5\frac{1}{8}$ | 19. $10\frac{3}{4}$ | 25. $8\frac{1}{2}$ |

PER CENTS AND COMMON FRACTIONS

86. Memorize this table; it contains the most important per cents and their equivalents in common fractions.

$$\begin{cases} 25\% = \frac{1}{4} \\ 50\% = \frac{1}{2} \\ 75\% = \frac{3}{4} \end{cases}$$

$$\begin{cases} 12\frac{1}{2}\% = \frac{1}{8} \\ 37\frac{1}{2}\% = \frac{3}{8} \\ 62\frac{1}{2}\% = \frac{5}{8} \\ 87\frac{1}{2}\% = \frac{7}{8} \end{cases}$$

$$\begin{cases} 14\frac{2}{7}\% = \frac{1}{7} \\ 71\frac{3}{7}\% = \frac{5}{7} \\ 1\% = \frac{1}{100} \\ 5\% = \frac{1}{20} \end{cases}$$

$$\begin{cases} 10\% = \frac{1}{10} \\ 20\% = \frac{1}{5} \\ 40\% = \frac{2}{5} \\ 60\% = \frac{3}{5} \\ 80\% = \frac{4}{5} \end{cases}$$

$$\begin{cases} 33\frac{1}{3}\% = \frac{1}{3} \\ 66\frac{2}{3}\% = \frac{2}{3} \\ 16\frac{2}{3}\% = \frac{1}{6} \\ 83\frac{1}{3}\% = \frac{5}{6} \\ 116\frac{2}{3}\% = \frac{7}{6} \end{cases}$$

$$\begin{cases} 100\% = 1 \\ 200\% = 2 \\ 120\% = \frac{6}{5} \\ 125\% = \frac{5}{4} \\ 133\frac{1}{3}\% = \frac{4}{3} \end{cases}$$

First Type Problem

87. To find a Per Cent of a Number

BY COMMON FRACTIONS

BY PERCENTAGE

1. How much is $\frac{3}{4}$ of \$160?

Process

$$\frac{3}{4} \text{ of } \$160 = \$120. \text{ Ans.}$$

2. How much is 60% of \$250?

Process

60% of a number = $\frac{3}{5}$ of the number.

$$\frac{3}{5} \text{ of } \$250 = \$150. \text{ Ans.}$$

BY DECIMAL FRACTIONS

3. How much is .37
of 240 mi.?

Process
240 mi.
.37
<hr/> 88.80 mi. <i>Ans.</i>

BY PERCENTAGE

4. How much is 53%
of \$865?

Process
\$865
.53
<hr/> \$458.45 <i>Ans.</i>

From these problems we see that to find a per cent of a number, we proceed as in finding a fractional part of a number.

88. In the question, How much is 60%
of \$250?

60% is called the **Rate** or **Rate per cent**.

\$250 is called the **Base**.

\$150 is called the **Percentage**.

The **Base** is the number on which the percentage is calculated (\$250).

The **Rate** or **Rate per cent** is the number of hundredths of the base to be taken (60)%.

The **Percentage** is the number obtained by taking the required per cent of the base (\$150).

The percentage is always of the same denomination as the base. Why?

89. To find the Percentage when the Base and Rate are given :

RULE. Multiply the base by the rate expressed as a common fraction or a decimal fraction.

EQUATION AND FORMULA.

$$\text{Percentage} = \text{Rate} \times \text{Base.} \quad P = R \times B.$$

NOTE. Whenever convenient, reduce the per cent to a common fraction in its lowest terms; then proceed as in common fractions. In other cases, reduce the per cent to a decimal fraction and proceed as in decimal fractions.

1. Find 20 % of \$ 15.30 ; of \$ 39.20 ; \$ 12.45.
2. Find $12\frac{1}{2}$ % of \$ 80 ; of \$ 240 ; of \$ 400.
3. Find $\frac{1}{8}$ % of \$ 80 ; of \$ 240 ; of \$ 400.
4. Find 25 % of \$ 64 ; \$ 200 ; of \$ 800.
5. Find $\frac{1}{4}$ % of \$ 64 ; \$ 200 ; of \$ 800.
6. Find 50 % of \$ 80 ; of \$ 120 ; of \$ 1000.
7. Find $\frac{1}{2}$ % of \$ 80 ; of \$ 120 ; of \$ 1000.
8. Find 200 % of 75 ; of 100 ; of 1500.
9. Find 150 % of 60 ; of 120 ; of 180.
10. Find $33\frac{1}{3}$ % of $\frac{1}{2}$; of $3\frac{1}{2}$; of $12\frac{1}{2}$.
11. $33\frac{1}{3}$ % of 60 + 75 % of 80 = ?
12. 75 % of 120 + $12\frac{1}{2}$ % of 120 = ?

DRILLS FOR LIMITED TIME

90. First: Begin at the top of a column and work down. Find values for all ?; (e.g. 10 % of 12 = ?; 10 % of 24 = ?; etc.).

Second: Begin at the left and work to the right. Find values for all ?; (e.g. 10 % of 12 = ?; 25 % of 12 = ?; etc.).

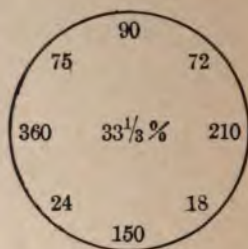
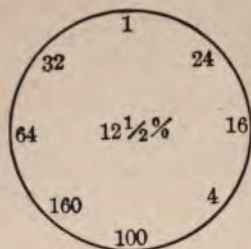
	BASE	10 %	25 %	33 $\frac{1}{3}$ %	50 %	75 %	100 %
1.	12	?	?	?	?	?	?
2.	24	?	?	?	?	?	?
3.	72	?	?	?	?	?	?
4.	60	?	?	?	?	?	?
5.	144	?	?	?	?	?	?
6.	1	?	?	?	?	?	?
7.	36	?	?	?	?	?	?
8.	120	?	?	?	?	?	?

91. Proceed as in Article 90.

	BASE	$\frac{1}{2}$ %	$\frac{1}{4}$ %	$\frac{1}{8}$ %	1 %	2 $\frac{1}{2}$ %	3 $\frac{1}{2}$ %
1.	100	?	?	?	?	?	?
2.	200	?	?	?	?	?	?
3.	400	?	?	?	?	?	?
4.	800	?	?	?	?	?	?
5.	1200	?	?	?	?	?	?
6.	1600	?	?	?	?	?	?
7.	3200	?	?	?	?	?	?
8.	4000	?	?	?	?	?	?

DRILLS FOR LIMITED TIME

92. The number in the center is the rate; each number in the ring is the base; find the percentage.



NOTE. Other devices suitable for drills in percentage may be found in *The Pupils' Arithmetic*, Book IV.

93.

WRITTEN EXERCISES

Find:

- | | |
|-----------------|-----------------------------------|
| 1. 26% of 950 | 12. 115% of 900 |
| 2. 18% of 260 | 13. 130% of \$1300 |
| 3. 33% of 175 | 14. 82% of 230 bu. |
| 4. 51% of 18 | 15. $14\frac{1}{2}\%$ of 150 ft. |
| 5. 24% of 150 | 16. $27\frac{1}{4}\%$ of \$200 |
| 6. 37% of 320 | 17. 32.8% of 110 |
| 7. 69% of 165 | 18. $1\frac{3}{4}\%$ of \$1000 |
| 8. 85% of 420 | 19. $\frac{1}{8}\%$ of \$2472 |
| 9. 43% of 275 | 20. $3\frac{1}{2}\%$ of \$1260.80 |
| 10. 97% of 1800 | 21. 34% of $750\frac{3}{5}$ |
| 11. 71% of 500 | 22. 145% of \$2500 |

94.

WRITTEN EXERCISES

1. Find 9% of 135; of 450; of 780; of $290\frac{1}{2}$.
2. Find 14% of 240; of 920; of 750; of 630.5.
3. Find 38% of \$27.50; of \$160; of \$930.50.
4. Find 95% of 1500 pupils; of 800 pupils;
of 2400 pupils.
5. Find $16\frac{1}{2}$ % of 80 ft.; of 560 bu.; of 3000 T.
6. Find 25.5% of \$1400; of 2500; of \$16,000.
7. Find 160% of \$300; of \$900; of \$1750.75.
8. Find 250% of \$700; of \$439.25; of
\$2670.30.
9. $3\frac{1}{2}$ % of 60 + $2\frac{1}{2}$ % of 60 = ?
10. $1\frac{1}{4}$ % of \$75 - $\frac{1}{2}$ % of \$75 = ?
11. 8% of 400 + 6% of 250 = ?
12. 12% of 150 - 7% of 250 = ?

95.

ORAL EXERCISES

- | | |
|---------------------|-----------------------|
| 1. Find 16% of 300. | 8. Find 9% of 100. |
| 2. Find 8% of 50. | 9. Find 11% of 150. |
| 3. Find 4% of 30. | 10. Find 15% of 60. |
| 4. Find 12% of 80. | 11. Find 2% of 150. |
| 5. Find 6% of 90. | 12. Find 1% of 350. |
| 6. Find 5% of 40. | 13. Find 100% of 425. |
| 7. Find 3% of 120. | 14. Find 300% of 125. |

- | | |
|------------------------------------|-----------------------------------|
| 15. Find $11\frac{1}{2}\%$ of 200. | 19. Find 200 % of 75. |
| 16. Find $21\frac{1}{2}\%$ of 100. | 20. Find $\frac{1}{2}\%$ of 800. |
| 17. Find 2 % of 400. | 21. Find $41\frac{1}{2}\%$ of 50. |
| 18. Find 18 % of 50. | 22. Find $\frac{1}{8}\%$ of 160. |

96.

ORAL EXERCISES

1. Find 25% of 40; of 80; of 100; of 1000.
2. Find 50% of 30 bu.; of 800 ft.; of \$420.
3. Find $33\frac{1}{3}\%$ of \$960; of 247 mi.; of 330 yd.
4. Find 60% of 120; of 200; of 10; of 75.
5. Find $66\frac{2}{3}\%$ of 99; of 180; of 300; of 1500.
6. Find 75% of 160 bbl.; of \$1600; of 240.
7. Find 150% of 200; of 320; of 40; of 100.
8. Find $37\frac{1}{2}\%$ of \$1.60; of \$4; of \$24.40.
9. Find 10% of \$17.30; of \$63.20; of \$89.20.

NOTE. To find 1%, point off two decimal places from the right (*e.g.* 1% of 185 = 1.85); if the base ends in two zeros, cancel the zeros (*e.g.* 1% of 500 = 5).

ORAL PROBLEMS

97. 1. Minneapolis has a population of 300,000 people. Oakland has a population 50 % as large. Find the population of Oakland.
2. A local train takes 15 hr. to go 240 mi. An express train takes only 40 % as long. How long does the express train take ?

3. George has 120 marbles. Harry has 125 % as many. How many marbles has Harry ?

4. The railroad fare between two cities was \$ 2.70. The company increased the fare $33\frac{1}{3}$ %. Find the increase in fare.

5. Meat which cost 16 ct. a pound has increased $37\frac{1}{2}$ % in price. How much more does a pound cost now ?

6. A company that employed 220 men discharged 20 % of them. How many men were discharged ?

7. A plot of ground is 440 ft. long. The width is $12\frac{1}{2}$ % of the length. Find the width.

8. A book cost \$ 1.20. It was sold for 80 % of its cost. For how much was it sold ?

9. The trunk of a tree was 35 ft. long. It was cut into two pieces, one being $14\frac{2}{7}$ % as long as the trunk. How long was each piece ?

10. 150 glass globes were shipped by express. 6 % of them were broken. How many were broken ?

11. A farmer had 250 cows. He sold 12 % of them. How many did he sell ?

12. There were 500 children in a school. 8 % were not promoted. How many were not promoted ?

13. A baseball team played 50 games. It won 90 % of them. How many games were won ?

WRITTEN PROBLEMS

98. 1. A school contains 3000 children. 54 % of them are boys. How many boys are there?

2. 40 words were given to each boy in an examination in spelling. George spelled 95 % of them correctly; Harry spelled 85 % of them correctly; Frank spelled 70 % correctly. How many words did George spell correctly? Harry? Frank?

3. A house is worth \$ 16,000. A second house is worth 92 % as much. Find the value of the second house.

4. During January, a store sold \$ 1500 worth of goods; during February, it sold 110 % as much. Find the value of the goods sold in February.

5. How many days are there in 78 % of a year? (365 da. = 1 yr.)

6. One piano cost \$ 550. A second piano cost 130 % as much. Find the cost of the second piano.

7. 20,000 copies of a book were sold during the first year. During the second year the sales increased 22 %. How many copies were sold during the second year?

8. Desks which were marked \$ 65 each were reduced 12 %.

(a) Find the reduction.

(b) How much is saved by buying 6 desks at the lower price?

9. A man who receives \$2000 a year saves 32 % of it. How much will he save in five years at that rate?

10. A man who receives \$3000 a year saves 35 % of it. How many years will it take to save \$4725 at that rate?

11. A number of books were packed in a box. The total weight of the books and the box was 65 lb. The box weighed 6 % of the total weight.

(a) Find the weight of the box.

(b) Find the weight of the books.

12. An orchard yielded 2500 bbl. of apples. 65% of them were sold at \$2.40 per barrel. The remainder were sold at \$2.70 per barrel. How much was received for all the apples?

13. A telephone company received 700 calls at one station. 95% of them were 5-ct. calls, the remainder were 10-ct. calls. How much money did the company receive for all the calls?

99. To find the Amount when the Base and the Rate are Given

WHEN THE RATE IS EASILY REDUCIBLE TO
A COMMON FRACTION

1. A boy who had 160 marbles won 25% as many. How many marbles has he now?

Process

Represent the original number of marbles by 100 %.

The number of marbles gained = $\frac{25}{100}$ % of the original number.

The number of marbles the boy has now = 125 %, or $\frac{5}{4}$, of the original number.

Therefore, $\frac{4}{5}$ of 160 = 200 marbles. *Ans.*

SECOND METHOD

$$\frac{1}{4} \text{ of } 160 = 40, \text{ increase; } 160 + 40 = 200.$$

**WHEN THE RATE IS NOT EASILY REDUCIBLE TO
A COMMON FRACTION**

2. Five years ago, the population of a city was 200,000. To-day the population is 23% greater. Find the population to-day.

Process

Represent the population 5 yr. ago by 100 %.

The increase in population = $\frac{23}{100}$ % of the population 5 yr. ago.

Therefore, the present population = 123 % of the population 5 yr. ago

$$1.23 \times 200,000 = 246,000. \quad \text{Ans.}$$

SECOND METHOD

$$.23 \text{ of } 200,000 = 46,000. \quad 200,000 + 46,000 = 246,000.$$

The answer in each is called the **Amount**.

Proof. Check the answers by Type 2 (p. 85) or by Type 3 Case 3 (p. 100).

TO FIND THE AMOUNT AND DIFFERENCE 81

100. RULE. To find the Amount: Add the percentage to the base, or multiply the base by (1 plus the rate).

EQUATION AND FORMULA.

Amount = Base + Percentage; $A = (B + P)$; or

Amount = Base \times (1 plus Rate); $A = B \times (1 + R)$.

101. To find the Difference when the Base and Rate are Given

WHEN THE RATE IS EASILY REDUCIBLE TO A COMMON FRACTION	WHEN THE RATE IS NOT EASILY REDUCIBLE TO A COMMON FRACTION
--	--

1. A boy who had 120 marbles lost 20% of them. How many marbles were left?	2. A merchant had 750 boxes of oranges. He sold 24% of them. How many boxes were left?
--	--

Process

Represent the original number of marbles by 100%.

The number of marbles lost = 20% of original number.

The number of marbles left = 80% or $\frac{4}{5}$ of the number boy had at first.

$\frac{4}{5}$ of 120 marbles = 96 marbles.
Ans.

Process

Represent the total number of boxes by 100%.

The number of boxes sold = 24% of total number.

Number of boxes not sold = 76% of total number.

.76 of 750 boxes = 570 boxes.
Ans.

SECOND METHOD

$\frac{1}{5}$ of 120 = 24 (loss);
120 - 24 = 96.

SECOND METHOD

.24 of 750 = 180 (sold);
750 - 180 = 570.

The answer in each is called the **Difference**.

102. RULE. To find the Difference: Subtract the percentage from the base; or multiply the base by $(1 - \text{rate})$.

EQUATION AND FORMULA.

Difference = Base - Percentage; $D = (B - P)$, or

Difference = Base \times $(1 - \text{Percentage})$; $D = B \times (1 - P)$.

Proof. Check the answers by Type 2 (p. 85), or by Type 3, Case 2 (p. 98).

ORAL PROBLEMS

103. 1. William weighed 60 lb. in June; in September, his weight was 10 % more. Find his weight in September.

2. One kind of tea costs 50 ct. a pound. A better kind costs 20 % more. Find the cost of 3 lb. of the better kind.

3. A wagon cost \$ 80. A horse cost 300 % as much as the wagon. Find the cost of the horse and the wagon.

4. Nellie had \$ 4. The first day she spent 50 % of it; the second day she spent 50 % of the remainder. How much did she spend the second day?

5. A has \$ 200; B has 50 % as much as A; C has 25 % as much as B. How much money has C?

6. The number of pupils in one school is 400; the number in a second school is 25 % greater; the number in a third school is 50 % greater than the number in the second school. Find the number of pupils in the third school.

WRITTEN PROBLEMS

104. 1. The cost of a railroad ticket was \$ 65. The railroad company increased the fare $3\frac{1}{2}\%$. Find the price of a ticket now.

2. The population of Albany in 1900 was about 94,000. In 1910, the population was 9% greater. Find the population in 1910.

3. A horse that cost \$ 350 was sold at a gain of 18%. How much was received for it?

4. 12,500 pr. of shoes were made in a factory in one year. The next year 12% more shoes were made. Find the number of pairs of shoes made during the second year.

5. A man who formerly paid \$ 45 a month for rent now pays 15% more. Find the amount paid for rent now.

6. The wages paid in a factory were as follows:

Skilled Workmen \$ 25 per week.

Unskilled Workmen \$ 15 per week.

Boys \$ 8 per week.

The skilled workmen received an increase of 25%, the unskilled workmen an increase of 15%, and the boys an increase of 7%. How much does each class receive per week after the increase?

7. The receipts of a store during the first week were \$ 350; the amount received during the sec-

ond week was 25 % greater than the amount received during the first week ; the amount received during the third week was 35 % greater than the amount received during the second week. Find the total amount received during the three weeks.

8. There are three grades of lead pencils. Grade A costs 5 ct. each ; grade B costs 20 % more than grade A ; grade C costs 50 % less than grade B. Find the cost of 6 doz. of grade C.

9. A tailor has 90 overcoats. He sells $33\frac{1}{3}\%$ of them at \$ 15 each ; 50 % of the remainder at \$ 18 each, and those still on hand at \$ 20 each. How much is received for all the overcoats ?

10. The annual salary of a clerk is \$ 1200. He spends 30 % for rent, 40 % for other expenses, and saves the remainder. How much does he save ?

11. A firm that employed 250 men discharged 12 % of that number.

(a) How many men are still employed ?

(b) How much money is paid to them if each man receives \$ 22 a week ?

12. One grade of butter costs 35 cents a pound ; another grade costs 40 % more. Find the cost of 5 lb. of the better quality.

13. 1500 people bought tickets for a theater. 48 % of them paid \$ 2 each ; 26 % paid \$ 1 each ; the remainder paid 75 ct. each. Find the total amount of money paid for tickets.

14. When a train started from Chicago it carried 200 passengers. At the first station 25% more passengers got aboard; at the second station, 40% of those in the train left; at the third station, 50% of those still aboard left. How many were still on the train?

Second Type Problem

105. To find what Per Cent one Number is of Another

1. A class contained 50 children. 10 of them were not promoted. What per cent of the class was not promoted?

The number not promoted equals 10 out of 50 or $\frac{10}{50}$ of the class.

$$\frac{10}{50} = \frac{1}{5} = 20\%. \quad \text{Ans.}$$

2. A storekeeper had 65 baseballs. He sold 51 balls. What per cent of his stock did he sell?

The number sold equals 51 out of 65 or $\frac{51}{65}$ of the entire stock.

$$\frac{51}{65} = 78\frac{6}{13}\%. \quad \text{Ans.}$$

106. RULE. To find the Rate: Divide the percentage by the base.

EQUATION AND FORMULA.

$$\text{Rate} = \frac{\text{Percentage}}{\text{Base}}; \left(R = \frac{P}{B} \right).$$

107. Proof. Check the answers by Type 1 (p. 70) or by Type 3, Case 1 (p. 93).

ORAL EXERCISE

108. What per cent is

- | | | |
|--------------|---------------|------------------|
| 1. 5 of 25? | 10. 2 of 8? | 19. 100 of 100? |
| 2. 10 of 50? | 11. 12 of 48? | 20. 250 of 500? |
| 3. 9 of 9? | 12. 8 of 32? | 21. 300 of 1000? |
| 4. 3 of 8? | 13. 3 of 4? | 22. 100 of 400? |
| 5. 4 of 4? | 14. 4 of 8? | 23. 250 of 1000? |
| 6. 3 of 10? | 15. 6 of 100? | 24. 740 of 1000? |
| 7. 4 of 20? | 16. 8 of 1? | 25. 150 of 600? |
| 8. 9 of 3? | 17. 9 of 6? | 26. 200 of 800? |
| 9. 6 of 2? | 18. 25 of 5? | 27. 250 of 1000? |
28. 5 is what % of 5? of 15? of 40?
29. 3 is what % of 1? of 8? of 15?
30. $\frac{1}{2}$ is what % of 1? of $1\frac{1}{2}$? of 4?
31. 5 is what % of 2? of 3.5? of 6?
32. 100 is what % of 20? of 50? of 25?

EXERCISES

109. 1. $4\frac{1}{2}$ is what per cent of 18?

Process. $\frac{4\frac{1}{2}}{18} = \frac{\frac{9}{2}}{18} = \frac{9}{2} \times \frac{1}{18} = \frac{1}{4} = 25\%$.

- | | |
|-------------------------|---------------------------------|
| 2. \$3 of \$2.50? | 5. 90 da. of $\frac{1}{2}$ da.? |
| 3. $1\frac{1}{2}$ of 5? | 6. 2 of $7\frac{1}{2}$? |
| 4. $3\frac{1}{2}$ of 6? | 7. 9 of $15\frac{1}{2}$? |

- | | |
|---------------------------|---------------------------|
| 8. $2\frac{1}{2}$ of 15? | 11. $5\frac{1}{2}$ of 25? |
| 9. $\frac{1}{2}$ of 8? | 12. 5 of 4.5? |
| 10. $3\frac{1}{2}$ of 10? | 13. 1.25 of 1? |

ORAL EXERCISES

110. What per cent is

- | | |
|-------------------------|-----------------------|
| 1. 1 ft. of a yard? | 11. 2 ft. of 2 yd.? |
| 2. 2 ft. of a yard? | 12. 6 in. of 2 ft.? |
| 3. 4 hr. of a day? | 13. 3 hr. of 1 da.? |
| 4. 12 hr. of a day? | 14. 3 hr. of 1 hr.? |
| 5. 1 da. of a week? | 15. 3 ft. of 8 ft.? |
| 6. 5 da. of a week? | 16. 2 yd. of 8 ft.? |
| 7. 3 in. of a foot? | 17. 40 cents of \$1? |
| 8. 9 in. of a foot? | 18. 70 cents of \$1? |
| 9. 20 min. of an hour? | 19. 1500 lb. of 1 T.? |
| 10. 40 min. of an hour? | 20. 3000 lb. of 1 T.? |

NOTE. Change both quantities to the same denomination before trying to find the rate.

DRILLS FOR LIMITED TIME

111. *First:* Begin at the top of a column and work down (*e.g.* 5 is what per cent of 5? 5 is what per cent of 10? 5 is what per cent of 20? etc.).

Second: Begin at the left and work to the right (*e.g.* 5 is what per cent of 5? 10 is what per cent of 5? 20 is what per cent of 5? etc.).

	BASE	5	10	20	40	50	100
1.	5	? %	? %	? %	? %	? %	? %
2.	10	? %	? %	? %	? %	? %	? %
3.	20	? %	? %	? %	? %	? %	? %
4.	30	? %	? %	? %	? %	? %	? %
5.	40	? %	? %	? %	? %	? %	? %
6.	50	? %	? %	? %	? %	? %	? %
7.	60	? %	? %	? %	? %	? %	? %
8.	80	? %	? %	? %	? %	? %	? %

112. Proceed as in Article 111.

	BASE	1	2	3	4	6	10
1.	1	? %	? %	? %	? %	? %	? %
2.	2	? %	? %	? %	? %	? %	? %
3.	3	? %	? %	? %	? %	? %	? %
4.	4	? %	? %	? %	? %	? %	? %
5.	5	? %	? %	? %	? %	? %	? %
6.	6	? %	? %	? %	? %	? %	? %
7.	8	? %	? %	? %	? %	? %	? %
8.	10	? %	? %	? %	? %	? %	? %

ORAL PROBLEMS

113. 1. Butter cost 35 ct. a pound. The price was increased 5 ct. Find the per cent of increase.

2. Meat which cost 20 ct. a pound, now costs 1. a pound. Find the per cent of increase.

3. A book was marked 80 ct. The price was reduced 10 ct. Find the per cent of reduction.

4. A book which was marked 90 ct. was sold for 60 ct. Find the per cent of reduction.

5. One package weighed 10 lb. Another package weighed 15 lb.

(a) The weight of the first package was what per cent of the weight of the second package?

(b) The weight of the second package was what per cent of the weight of the first package?

6. A boy who sold newspapers began with \$ 2. At the end of the day he had \$ 4. What per cent of his original amount had he gained?

7. There were 60 books on a shelf. 10 were arithmetics; the rest were readers.

(a) What per cent of the books were arithmetics?

(b) What per cent of the books were readers?

8. A book that cost 80 ct. is sold for 90 ct. What per cent of the cost is gained?

9. George receives \$ 8 a week; Harry receives \$ 4 a week. George's salary is what per cent of Harry's salary?

10. 3 toys cost 20 ct. each. They are sold for 25 ct. each.

(a) What per cent of the cost is gained on 1 toy?

(b) What per cent of the cost is gained on the 3 toys?

NOTE. Always calculate the rate per cent on the base.

11. 35 pupils of a class are present. 5 are absent. What per cent of the class is present? What per cent is absent?

12. Handkerchiefs that cost \$ 4 a dozen are sold at 50 ct. each. Find the per cent profit.

WRITTEN PROBLEMS

114. 1. In a class of 43 pupils, 39 are promoted.

(a) What per cent are promoted?

(b) What per cent are not promoted?

2. A plot of ground is 75 ft. long and 175 ft. wide.

(a) The length is what per cent of the width?

(b) The width is what per cent of the length?

3. Mr. Jones, Mr. Frank, and Mr. Williams contribute to a fund. Mr. Jones contributes \$ 150, Mr. Frank, \$ 175, and Mr. Williams, \$ 225.

(a) What per cent of the total amount is contributed by Mr. Jones? (b) By Mr. Williams?

4. A street is 400 ft. long and 20 yd.* wide. The length is what per cent of the width?

5. 1500 ft. is what per cent of 1 mi.?

6. The population of a town increased from 2500 to 3400. Find the per cent of increase.

7. Three years ago the population of a town was 3000. To-day the population is 750 greater. Find the per cent of increase.

*NOTE. Reduce all terms to the same denomination before finding the rate per cent.

8. A man who is paid at the rate of \$ 30 a week received \$ 17.50. What per cent of the weekly wage did he receive ?

9. \$ 1000 is divided equally among 15 children. What per cent of the sum is given to each child ?

10. I buy 2 doz. picture post cards at 1 ct. each and sell them at 2 ct. each. What per cent of the cost is gained ?

11. Shoes that were marked at \$ 4 a pair were sold for \$ 2.50 a pair.

(a) Find the per cent of reduction on 1 pr.

(b) Find the per cent of reduction on 3 pr.

12. A baseball club won 104 games and lost 16 games. What per cent of the games did the club win ?

13. Mr. Daly receives a salary of \$ 3000. He spends 25 % of it for rent, \$ 600 for other expenses, and saves the rest. What per cent of his salary does he save ?

14. Mr. Wood had 250 baseballs. He sold 20 % of them the first day and 40 % of the remainder the second day. What per cent of his original stock was left ?

15. A school contains 750 boys and 650 girls.

(a) The boys form what per cent of the total register ?

(b) The girls form what per cent of the total register ?

Third Type Problem (Case 1)

115. To find the Base when the Percentage and the Rate are Given

BY COMMON FRACTIONS

1. $\frac{3}{8}$ of the number of boys in a class is 15. How many boys are in the entire class?

Process

$\frac{3}{8}$ of the no. in the class = 15.

$\frac{1}{8}$ of the no. in the class = 5.

$\frac{8}{8}$ or the no. in the class = 40.

Or (shorter method),

$$15 \div \frac{3}{8} = 15 \times \frac{8}{3} = 40. \text{ Ans.}$$

BY PERCENTAGE

2. $37\frac{1}{2}\%$ of the number of boys in a class is 15. How many boys in the entire class?

Process

$37\frac{1}{2}\%$ of the number in class = 15.

Therefore, $\frac{3}{8}$ of the number in the class equals 15.

The solution is the same as in problem 1.

The number in class =

$$15 \div \frac{3}{8}, \text{ or } 15 \times \frac{8}{3} = 40. \text{ Ans.}$$

BY DECIMAL FRACTIONS

3. .28 of the distance traveled by a boat is 560 mi. Find the entire distance traveled by the boat.

Process

.28 of the distance equals 560 mi. Therefore, the total distance equals

$$560 \div .28 = 2000. \text{ Ans.}$$

BY PERCENTAGE

4. 28 % of the distance traveled by a boat is 560 mi. Find the entire distance traveled by the boat.

Process

$$28\% = .28$$

Therefore, the solution is the same as in problem 3.

$$560 \div .28 = 2000 \text{ mi. Ans.}$$

116. RULE. To find the Base: Divide the percentage by the rate.

EQUATION AND FORMULA.

$$\text{Base} = \frac{\text{Percentage}}{\text{Rate}}; B = (P \div R).$$

117. Proof. Check the answers by Type 1 (p. 70), or by Type 2 (p. 85).

ORAL EXERCISES

118. Find the number if

1. 50 % of it = 5
2. 10 % of it = 15
3. $12\frac{1}{2}$ % of it = 6
4. $16\frac{2}{3}$ % of it = 10
5. $87\frac{1}{2}$ % of it = 14
6. 75 % of it = 9
7. $33\frac{1}{3}$ % of it = 9
8. 20 % of it = 8
9. $14\frac{2}{7}$ % of it = $2\frac{1}{2}$
10. $37\frac{1}{2}$ % of it = 6
11. 50 % of it = $4\frac{1}{2}$
12. 150 % of it = 12
13. 5 is 20 % of what no.? 40 % of what no.?
14. 9 is 30 % of what no.? 50 % of what no.?
15. 6 is 75 % of what no.? $12\frac{1}{2}$ % of what no.?
16. $3\frac{1}{2}$ is 25 % of what no.? 10 % of what no.?
17. $\frac{1}{4}$ is $12\frac{1}{2}$ % of what no.? 50 % of what no.?
18. $\frac{3}{4}$ is $33\frac{1}{3}$ % of what no.? 25 % of what no.?
19. $\frac{2}{3}$ is 50 % of what no.? $66\frac{2}{3}$ % of what no.?
20. 60 is $16\frac{2}{3}$ % of what no.? 75 % of what no.?
21. 48 is 150 % of what no.? 200 % of what no.?
22. 150 is 300 % of what no.? 100 % of what no.?

DRILLS FOR LIMITED TIME

119. First: Begin at the top of a column and work down; (e.g. 3 is 10 % of ? 4 is 10 % of ? 5 is 10 % of ? etc.).

Second: Begin at the left and work to the right; (e.g. 3 is 10 % of ? 3 is 20 % of ? 3 is 25 % of ? etc.).

	PERCENT- AGE	10 % of	20 % of	25 % of	50 % of	33 $\frac{1}{3}$ % of	16 $\frac{2}{3}$ % of
1.	3 is	?	?	?	?	?	?
2.	4 is	?	?	?	?	?	?
3.	5 is	?	?	?	?	?	?
4.	6 is	?	?	?	?	?	?
5.	8 is	?	?	?	?	?	?
6.	10 is	?	?	?	?	?	?
7.	12 is	?	?	?	?	?	?
8.	20 is	?	?	?	?	?	?

WRITTEN EXERCISES

120. Find the number if

- | | |
|------------------------------------|---------------------------------------|
| 1. 85% of it = 680 | 9. 2% of it = 7.20 |
| 2. 36% of it = 612 | 10. 45% of it = 686.25 |
| 3. 71 $\frac{3}{4}$ % of it = 2500 | 11. 21% of it = 264.60 |
| 4. 55 $\frac{5}{8}$ % of it = 430 | 12. 38 $\frac{1}{2}$ % of it = 1617 |
| 5. 33% of it = 297 | 13. 64 $\frac{1}{2}$ % of it = 322.5 |
| 6. 16% of it = 29.6 | 14. 25 $\frac{1}{4}$ % of it = 176.75 |
| 7. 93% of it = 255.75 | 15. 15 $\frac{1}{4}$ % of it = 305 |
| 8. 51% of it = 187.48 | 16. 128% of it = 960 |

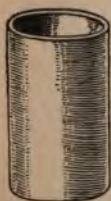
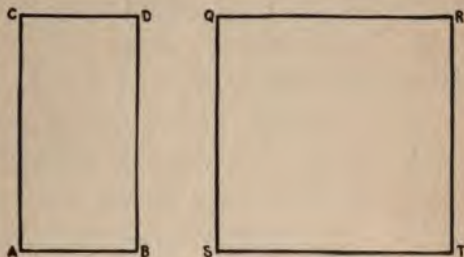
ORAL PROBLEMS

121. 1. The line AB , which is 25% of AC , is 4 in. long. How long is AC ?



2. The area of $ABCD$ is 50% of the area of $QRST$. If $ABCD$ contains 125 sq. in., find the area of $QRST$.

HINT. Indicate as nearly as possible on $QRST$ the area of $ABCD$.

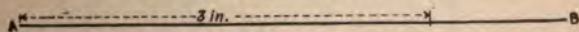


3. A boy can walk 4 blocks in $12\frac{1}{2}\%$ of an hour. How many blocks can he walk in an hour?

4. 20% of the water in this jar is 2 qt. How many quarts does the jar hold?

HINT. Indicate as nearly as possible the height of water in the jar.

5. 75% of the distance from A to B is 3 in. Find the total distance from A to B .



6. Harry has 20 ct. This is 25% of the amount George has. How much has George?

7. William has 50 ct. This is 200% of the amount John has. How much has John?

8. $12\frac{1}{2}\%$ of the number of boys in a class is 5. How many boys are in the class?

9. Find the height of a flagpole if $16\frac{2}{3}\%$ of the height equals 6 ft.

10. The width of a room is $66\frac{2}{3}\%$ of the length. The width is 20 ft. Find the length.

11. The width of a room is $66\frac{2}{3}\%$ of the length. The width is 20 ft. Find the length.

12. A grocer has two kinds of tea. The cheaper kind costs 60% of the price of the better kind. If the cheaper kind costs 45 ct. a pound, find the cost of a pound of the better kind.

13. The number of games lost by a club equals $14\frac{2}{7}\%$ of the number won. If the club lost 7 games, how many games did it win?

14. $87\frac{1}{2}\%$ of the length of a rope equals 14 ft. How long is the rope?

15. A book is sold for 80 ct. This is 125% of the cost. Find the cost.

WRITTEN PROBLEMS

122. 1. A train travels 10.8 mi. in 18% of an hour. How far does it travel in 1 hr.?

2. The height of a flagpole is 78% of the height of a building. If the pole is 27.3 ft. high, how high is the building?

3. There are 1000 pupils in one school. This is 250% of the number in a second school. How many pupils are there in the second school?

4. The population of a city is 288,000. This is 72% of the population of a second city. Find the population of the second city.

5. 92% of the number of pupils in a school equals 2024. How many pupils are there in the school?

6. Two men, A and B, were candidates for an office. A received 7500 votes. This was 150% of the number B received.

(a). Who was elected, A or B? Why?

(b). How many votes did B receive?

7. The population of Buffalo is 190% of the population of Rochester. The population of Buffalo is about 425,600.

(a) Which city has the larger population?

(b) Find the population of Rochester.

8. A printer has an order to print 1750 cards. If it takes 5 hr. to print 65% of them, how long will it take to print all the cards?

9. 12,000 writing pencils were sold. This is 120% of the number of drawing pencils sold. How many drawing pencils were sold?

10. If it requires $14\frac{1}{2}$ da. to build 85% of a wall, how many days will be required to build the entire wall?

11. By going at the rate of $38\frac{1}{4}$ mi. per hour for 12 hr., a train traveled 85 % of the distance between two cities. Find the total distance between the cities.

Third Type Problem (Case 2)

123. To find the Base when the Difference and the Rate of Decrease are Given

WHEN THE RATE IS EASILY REDUCIBLE TO A
COMMON FRACTION

1. After selling 20 % of his stock, a storekeeper has 48 books left. How many books had he at first?

Process

Represent the original number of books by 100 %.

Number of books sold $\qquad\qquad\qquad = 20\%$ of original
number.

Number of books left $\qquad\qquad\qquad = 80\%$ of original
number.

Therefore, 80 % or $\frac{4}{5}$ of the original number = 48.

The original number $= 48 \div \frac{4}{5} = 48 \times \frac{5}{4} = 60$ books. *Ans.*

WHEN THE RATE IS NOT EASILY REDUCIBLE TO A
COMMON FRACTION

2. A farmer sent a number of barrels of apples to the city. 18 % of them were destroyed. The remainder, 410 bbl., were in good condition. How many barrels were sent by the farmer?

Process

Represent the total number of
 barrels by 100%
 The number destroyed $= 18\%$ of the total number.
 The number in good condition $= 82\%$ of the total number.
 Therefore, 410 bbl. $= 82\%$ of the total number.
 Therefore, total number $= 410 \div .82 = 500$ bbl. *Ans.*

124. RULE. To find the Base: Divide the difference by 1 minus the rate.

EQUATION AND FORMULA.

$$\text{Base} = \text{Difference} \div (1 - \text{Rate}),$$

$$B = D \div (1 - R).$$

125. Proof. Check the answers by Type 1 (p. 70, to find the difference), or by Type 2 (p. 85).

Third Type Problem (Case 3)

126. To find the Base when the Amount and the Rate of Increase are Given

WHEN THE RATE IS EASILY REDUCIBLE TO A
 COMMON FRACTION

1. A boy's salary was increased to \$14. This is $16\frac{2}{3}\%$ more than he received last week. How much did he receive last week?

Process

Represent original salary by 100% .
 The increase in salary $= 16\frac{2}{3}\%$ of original salary.
 The new salary $= 116\frac{2}{3}\%$ of original salary.
 Therefore, $116\frac{2}{3}\%$ or $\frac{7}{6}$ of the original salary $= \$14$.

$$\text{Original salary} = \$14 \div \frac{7}{6} = \$14 \times \frac{6}{7} = \$12. \quad \text{Ans.}$$

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Or

Represent original salary by $\frac{6}{6}$.Increase in salary $= \frac{1}{6}$ of original salary.New salary $= \frac{7}{6}$ of original salary.Original salary $= \$14 \div \frac{7}{6} = \12 . *Ans.*

WHEN THE RATE IS NOT EASILY REDUCIBLE TO A
COMMON FRACTION

2. A man who had a sum of money in the bank increased his savings by 15 %. He then had \$ 345 in the bank. How much had he at first?

Process

Represent the original amount in
the bank by

100 %.

Then the increase

 $= 15\%$ of the original
amount.

And the total amount on deposit

 $= 115\%$ of the original
amount.Therefore, the original amount is $\$345 \div 1.15 = \300 . *Ans.*

127. RULE. To find the Base: Divide the amount by 1 plus the rate.

EQUATION AND FORMULA.

Base = Amount \div (1 + Rate); $B = A \div (1 + R)$.

128. Proof. Check the answers by Type 1 (p. 79, to find the amount), or by Type 2 (p. 85).

ORAL PROBLEMS

129. 1. Henry has 30 marbles. This is 50 % more than he had yesterday. How many did he have yesterday?

2. Mary spent 20 % of her money. She then had 20 ct. left. How much had she at first ?

3. On promotion day, 10 % of the pupils were not promoted. If 45 pupils were promoted, what was the total number before promotion ?

4. A boy sold $12\frac{1}{2}$ % more papers to-day than yesterday. He sold 45 papers to-day. How many did he sell yesterday ?

5. The price of butter was increased 10 %. If butter costs 33 ct. per pound now, what did it cost before the increase in price ?

6. A boy's salary was increased to \$ 9 a week. This was an increase of 20 %. What was the salary before the increase ?

7. On Monday, a storekeeper had a number of books. On Tuesday, he had only 34 books left. This was $66\frac{2}{3}$ % less than the number he had on Monday. How many books had he sold ?

8. One quality of coffee cost 35 ct. a pound. This is $12\frac{1}{2}$ % cheaper than a better quality. Find the cost of a pound of the better quality.

9. A statue and its base weigh 80 lb. This is 25 % more than the weight of the statue. Find the weight of the statue.

10. A boy weighed 120 lb. in September. This was $14\frac{2}{7}$ % more than he weighed in June. How much did he weigh in June ?

WRITTEN PROBLEMS

130. 1. The price of a suit of clothes this year is \$ 27.25, which is 9 % greater than the price last year. Find the price last year.

2. The area of Mr. Howell's farm is 42 A. This is 16 % less than the area of Mr. Holt's farm. Find the area of Mr. Holt's farm.

3. 14 % of the length of a log is 3.36 ft. Find the length of the log.

4. The price of a pound of butter is 24 % greater than it was last month. Find the cost of 5 tubs, each containing 52 lb., at last month's price, if the present price of butter is 31 ct. per pound.

5. The population of Massachusetts increased 20 % during the last 10 yr. If the present population is 3,366,000, what was the population 10 yr. ago?

6. The population of Iowa decreased .3 % during the last 10 yr. If the present population is 2,134,000, find the decrease during the last 10 yr.

7. The ocean steamship *Olympic* is 882 ft. long. This is 11.7 % more than the length of the *Lusitania*. How much longer is the *Olympic* than the *Lusitania*?

8. In 1911, a company sold 20 % more lamps than in 1910. In 1912, the company sold 20 % more lamps than in 1911. If 720 lamps were sold in 1912,

(a) How many lamps were sold in 1911?

(b) How many lamps were sold in 1910?

9. A horse was sold for \$ 595. This was a loss of 15 % on the cost. Find the cost.

10. After using 84 % of his flour, a baker had 40 bbl. left.

(a) How many barrels had he at first?

(b) How many barrels did he use?

11. A number of boys took part in a race. After a while 25 % of them dropped out. Later, 40 % of the remainder dropped out. There were 18 boys left. How many started in the race?

12. The number of houses sold in a city in 1912 was 20 % less than the number sold in 1911; but the number sold in 1911 was 10 % greater than the number sold in 1910. If the number sold in 1912 was 1056,

(a) How many were sold in 1911?

(b) How many were sold in 1910?

13. The number of boys in a school is 50 % greater than the number of girls. The number of boys is 1440. Find the number of girls.

MISCELLANEOUS PROBLEMS

(Use pencil and paper only when necessary.)

131. 1. One building is 700 ft. high. A second building is 82 % as high. How high is the second building?

2. In a school containing 850 pupils, 824 are present.

(a) What per cent are present?

(b) What per cent are absent?

3. If a train travels 38 % of the distance between two cities in an hour, how long will be required to travel the entire distance?

City	Approx. Pop. 1900	Increase in 10 yr.	Pop. in 1910
4. Cincinnati	325,500	11.6 %	?
5. Cleveland	381,700	47 %	?
6. New York	3,437,000	38.7 %	?
7. Chicago	1,698,500	38.8 %	?
8. Denver	133,500	59.4 %	?

9. Best quality tea costs 56 ct. per pound. This is 12 % more than the cost of the second quality. Find the cost of the second quality per pound.

10. Three men were candidates for an office. A received 2500 votes; B, 3500; C, 4000 votes.

(a) B's vote was what per cent of C's vote?

(b) What per cent of the total vote was cast for A?

11. After having traveled 47 % of the distance between two cities, a train still had 91 mi. to go. Find the distance between the cities.

12. Mr. Jordan spent \$ 1500. 32 % was spent for a horse, 15 % for a wagon, and the rest for tools. How much was spent for tools?

13. 58 desks were bought for \$ 25 each. They were sold at a gain of 16 %. How much was received for them?

14. 64 % of the cost of a desk is \$ 115.20. Find the cost of the desk.

15. One river is 950 mi. long. Another river is 42 % as long. How long is the second river?

16. An ocean steamship is 800 ft. long. This is $114\frac{2}{3}$ % of the height of the Singer building in New York. How high is the Singer building?

17. 230 da. is what per cent of 1 yr. ($365\frac{1}{4}$ da.)?

18. \$ 15.50 is what per cent of \$ 75?

19. The population of Virginia increased 11.2 % during the last 10 yr. If the present population is 2,168,400, find the increase during the last 10 yr.

20. 82 % of the water in a tank weighs 205 lb. Find the weight of all the water in the tank.

21. How many feet are there in 62 % of a mile?

22. A manufacturer of shoes made 350 pr. of children's shoes, and 120 % as many pairs of ladies' shoes. How many pairs of ladies' shoes were made?

23. A company employed 220 men during the regular season. During the busy season it increased its force by 35 %. How many men were employed during the busy season?

24. A factory made 1189 gross of pencils during June; this was 18 % less than the number made during May. Find the total number of pencils made during May and June.

25. The expenses of a manufacturer are 92 % of his receipts. The expenses are \$5704. Find the receipts.

26. 40 words were given in an examination in spelling. George spelled 40 correctly; Mary spelled 38 correctly; Harry, 35 correctly. Find the per cent of each pupil.

	Register	No. Present	Per Cent Present	Per Cent Absent
27. First grade	42	39	?	?
28. Second grade	41	40	?	?
29. Third grade	45	43	?	?
30. Fourth grade	44	44	?	?

31. In a school containing 1800 children, 3 children were late in the morning and 2 children were late in the afternoon.

(a) What per cent of the children were late in the morning?

(b) What per cent of the children were late in the afternoon?

32. A piece of cloth contained 30 yd. 75% of it was sold at \$2 a yard; the remainder was sold at \$3 a yard. How much was received for the entire piece of cloth?

33. During July the receipts of a store were \$2750; during August, \$2250. Find the per cent of decrease.

34. A school contains 800 boys. This is 125% of the number of girls.

(a) Are there more girls or boys in the school?

(b) Find the number of girls.

35. A manufacturer spent \$12,000. 74% of this money was paid to his employees. The remainder was spent for materials.

(a) How much was paid to the employees?

(b) How much was spent for materials?

36. The workmen in a factory obtained an increase of 9% in wages. If the owners of the factory pay \$2997.50 in wages after the increase, how much did they pay before the increase in wages?

37. The number of pupils present in school is 96% of the number on register. How many pupils are on register, if the number present is 720?

38. In one year a field produced 900 bu. of wheat. The next year it produced 175 bu. less. Find the per cent of decrease.

39. 12,160 persons or 8 % of the population of a city were born in a foreign country. Find the population of the city.

40. In 1900 the population of a city was 375,000. In 1910 the population was 22 % greater. Find the increase in population.

41. A man who receives \$ 2500 a year spends 23 % of it for rent, 45 % for food and clothing, 12 % for other expenses, and saves the remainder.

(a) How much is spent for rent?

(b) How much is spent for food and clothing?

(c) How much is saved ?

42. Pianos which were marked at \$ 525 each were reduced 12 %. Find the reduced price.

43. A house was sold for \$ 575. This was a gain of 15 % on the cost. Find the cost.

44. The profits of a business man in 1912 were \$ 5200. This was $8\frac{1}{3}$ % greater than the profits in 1911, and 30 % greater than the profits in 1910.

(a) Find the profits in 1911.

(b) Find the profits in 1910.

45. A ton of coal is put into 35 bags, each containing an equal amount. What per cent of the ton is in each bag?

46. A post-office clerk sold the following :

1000 one-cent stamps.

30 % more two-cent stamps than one-cent stamps.

20 % less post cards (1 cent each) than two-cent stamps.

(a) How much was received for the post cards ?

(b) How much was received in all ?

47. I buy 4 doz. picture post cards at 1 ct. each and sell them at 2 ct. each. What per cent of the cost is gained ?

48. The value of a building is \$ 38,500. This is 23 % less than the value of an adjoining building. Find the value of the second building.

49. 500 people were employed in a factory. 72 % of them were men ; 22 % of them were women ; the rest were children. How many men were employed ? How many women ? How many children ?

50. A merchant bought 150 overcoats. He paid \$ 12 each for 40 % of them and \$ 15 each for the remainder. How much did he pay for all the overcoats ?

51. Apples that cost 2 for 1 ct. are sold at 1 ct. each. What per cent of the cost is gained ?

52. Oranges that cost 1 ct. each are sold at the rate of 3 for 5 ct. What per cent of the cost is gained ?

53. Lead pencils that cost 20 ct. a dozen are sold for 2 ct. each.

(a) What per cent is gained on 1 doz. ?

(b) What per cent is gained on 3 doz. ?

54. A man who desired to buy a house saved \$ 850 a year for 6 yr. He thus saved 68 % of the cost of the house. How much more money must he save to buy the house?

55. By sailing at the rate of 30 mi. an hour, a boat completed $56\frac{1}{4}$ % of its journey in $4\frac{1}{2}$ hr. How many more miles must the boat sail to complete the whole distance?

56. A baseball team lost 35 % of its games. If it won 78 games, how many games were lost?

57. 60 % of the population of one town equals 80 % of the population of a second town. If the population of the second town is 3600, find the population of the first town.

COMMISSION

FOR READING AND DISCUSSION

132. Commission may be earned in various ways.

1. A firm that has a large number of customers may employ collectors to collect bills that are due. These men may be paid a commission on the amount they collect.

2. Agents are often employed to buy or sell goods; *e.g.* a contractor may employ an agent to buy 100 teams of horses; or a firm may employ agents to sell goods. These agents are paid a percentage of the value of the goods bought or sold.

3. Agents, called brokers, are employed to buy and sell real estate. They are paid a percentage of the value of the property bought or sold. The money they receive is usually called **brokerage**. Salesmen or agents are sometimes paid a commission on the amount of goods they sell.

4. Agents who buy or sell stocks or bonds, who draw up insurance policies, who procure loans, charge a commission or brokerage. Commissions are paid also to architects, builders, contractors, auctioneers, and other persons who render valuable services to their principals.

To find the Commission or Brokerage

133. 1. Mr. Graham sells \$400 worth of goods for Mr. Brown. His rate of commission is 6%.

(a) How much is his commission?

(b) How much does he send to Mr. Brown?

2. A contractor orders his agent to buy 400 horses at \$75 each. The agent charges 4% commission.

(a) Find the commission.

(b) How much does the contractor pay for the horses?

Process

\$ 400	Gross Proceeds
.06	Rate of Commission
<u>\$ 24.00</u>	Commission
\$ 376.00	Net Proceeds

\$400 is called the **gross proceeds** of the sale, and \$376 is called the **net proceeds**.

Process

\$ 75	
<u>400</u>	
\$ 30000	
.04	
<u>\$ 1200.00</u>	
\$ 31,200.00	Total Cost

\$30,000 is called the **net cost** and \$31,200 is called the **gross cost**.

Commission or brokerage is always calculated on the value of the goods bought or sold.

The method of finding the commission is similar to the method of finding a per cent of a number (First Type Problem, p. 70).

134. The **Value of the Goods** corresponds to the **Base**.

The **Rate of Commission**
The **Rate of Brokerage** } corresponds to the
Rate Per Cent.

The **Commission** or **Brokerage** corresponds to the **Percentage**.

The **Amount remitted** (called **Net Proceeds**) corresponds to the **Difference**. (See problem 1.)

The **Total** or **Gross Cost** (in problem 2) corresponds to the **Amount**.

135. RULE. To find the **Commission** or **Brokerage** :
Multiply the value of the goods bought or sold by the rate expressed as a decimal or a common fraction.

ORAL EXERCISES

136. Find the commission in each of the following:

Value of Goods	Rate of Commission	Value of Goods	Rate of Commission
1. \$ 800	10 %	4. \$ 1200	5 %
2. \$ 1000	10 %	5. \$ 250	4 %
3. \$ 500	5 %	6. \$ 400	4 %

Value of Goods	Rate of Commission	Value of Goods	Rate of Commission
7. \$ 5000	2 %	14. \$ 2000	2 %
8. \$ 600	$1\frac{1}{2}\%$	15. \$ 5000	$1\frac{1}{2}\%$
9. \$ 900	1 %	16. \$ 10,000	$2\frac{1}{2}\%$
10. \$ 1400	$\frac{1}{2}\%$	17. \$ 2500	5 %
11. \$ 1500	6 %	18. \$ 1850	5 %
12. \$ 700	3 %	19. \$ 4000	$\frac{1}{8}\%$
13. \$ 1640	$12\frac{1}{2}\%$	20. \$ 1600	$\frac{1}{8}\%$

WRITTEN PROBLEMS

137. 1. A firm employs Mr. A to collect bills. On Monday he collects \$ 250; on Tuesday, \$ 400; on Wednesday, \$ 150; on Thursday, \$ 500; on Friday, \$ 100; and on Saturday, \$ 50. Find Mr. A's commission, if he receives $1\frac{1}{2}\%$ of the amount he collects.

2. An agent sells \$ 250 worth of apples for a farmer. He charges 5% commission.

(a) How much does he receive as commission?

(b) How much does he remit to the farmer?

3. An agent buys \$ 500 worth of goods for Mr. B and charges $3\frac{1}{2}\%$ commission.

(a) Find his commission.

(b) Find the gross cost.

4. An agent sold a house for \$ 15,000. He charged $2\frac{1}{2}\%$ commission. Find his commission and the amount of money he sent to the owner.

5. An agent sold 60 lots at \$ 500 a lot. Find his commission at $2\frac{1}{2}\%$.

6. Mr. Jones sent his agent 200 bbl. of apples, 300 bbl. of potatoes, and 150 bbl. of cabbages.

The agent sold the apples at \$ 2.75 per barrel, the potatoes at \$ 1.50 per barrel, and the cabbages at 85 ct. per barrel.

(a) Find his commission at 4 %.

(b) How much did Mr. Jones receive ?

7. Mr. Smith sent his agent 450 lb. of poultry, 75 crates of eggs, and 150 lb. of butter.

The agent sold the poultry at 19 ct. per pound, the eggs at \$ 1.44 per crate, and the butter at 22 ct. per pound.

Find the agent's commission at 4 %. Find the net proceeds.

8. Mr. Brown sent the following to his agent: 150 bbl. of apples, 225 bbl. of potatoes, 75 bbl. of turnips. The agent sold $\frac{1}{2}$ the apples at \$ 2.50 per barrel and the remainder at \$ 3 per barrel; the potatoes at \$ 1.75 per barrel, and the turnips at \$ 1.25 per barrel.

Find the agent's commission at 6 %. Find the net proceeds.

9. Mr. Herman is a salesman for a sewing machine company. He receives a commission of 6 % on all the machines he sells. During the first week he sold 10 machines ; during the second

week, 14 machines; during the third week, 20 machines.

Find his total commission for the three weeks if the machines were sold at \$32 each.

10. Mr. Williams is employed by a book company. He receives a salary of \$30 a week and a commission of 10 % on all sales. During the first week he sold 20 books; during the second week, 35 books; during the third week, 50 books.

Find the total amount earned by Mr. Williams in the three weeks, if the books were sold at \$2.50 each.

To find the Rate of Commission or Brokerage

138. An agent sells \$500 worth of goods and retains \$10 for his commission. Find the rate of commission.

The rate is always calculated on the value of the goods bought or sold.

Process

$$\begin{aligned} \text{The rate per cent} &= \frac{10}{500} = \frac{2}{100} \\ &= 2\%. \end{aligned}$$

An agent who buys \$1250 worth of goods charges \$37.50 for his services. Find the rate of commission.

Process

$$\begin{aligned} \text{The rate per cent} &= \frac{37.50}{1250} = \\ 37.50 \div 1250 &= .03 = 3\%. \end{aligned}$$

The method of finding the rate of commission or brokerage is similar to the method of finding what per cent one number is of another (Second Type Problem, p. 85).

139. To find the Rate of Commission or Brokerage:

RULE. Divide the commission or brokerage by the value of the goods bought or sold.

EXERCISES

140. Find the rate of commission in each of the following :

	Value of Goods	Commission		Value of Goods	Commission
1.	\$ 200	\$ 40	11.	\$ 500	\$ 10
2.	\$ 400	\$ 40	12.	\$ 600	\$ 3
3.	\$ 50	\$ 10	13.	\$ 800	\$ 20
4.	\$ 250	\$ 10	14.	\$ 300	\$ 10.5
5.	\$ 1000	\$ 20	15.	\$ 640	\$ 80
6.	\$ 60	\$ 3	16.	\$ 1000	\$ 25
7.	\$ 320	\$ 80	17.	\$ 75	\$ 17
8.	\$ 120	\$ 20	18.	\$ 950	\$ 9.5
9.	\$ 400	\$ 4	19.	\$ 800	\$ 2
10.	\$ 600	\$ 9	20.	\$ 1600	\$ 2

WRITTEN PROBLEMS

141. 1. An agent collected bills amounting to \$ 5000 and received \$ 125 for his commission. Find his rate of commission.

2. A commission merchant sold goods valued at \$ 750. Find his rate of commission, if he received \$ 37.50 for his services. Also find the amount remitted to the owner.

3. A real estate broker sold a house for \$ 20,000. His brokerage was \$ 300. Find the rate of brokerage or commission. How much did the owner receive?

4. An agent sold two houses, one for \$15,000 and one for \$17,500. What rate of commission did he charge, if he received \$1625?

5. An agent sold 40 sets of books, each set containing 6 vols., at \$3 a volume. Find the rate of commission, if the agent received \$43.20.

6. A salesman sold 20 tables at \$25 each and 120 chairs at \$1.50 each. Find the rate of commission, if the salesman received \$27.20.

7. A salesman is paid a weekly salary of \$25 and a commission on all the tables he sells. He sells 10 tables at \$35 each in one week. Find the rate of commission paid him if his total earnings during the week were \$40.

To find the Value of Goods bought or sold when the Commission and the Rate are Given

142. 1. An agent receives $2\frac{1}{2}\%$ commission for selling a piece of land. Find the value of the land if the agent's commission is \$500.

Process. \$500 is $2\frac{1}{2}\%$ of the value of the land.

Therefore, the value of the land = $\$500 \div .02\frac{1}{2} = \$20,000$.

The method of finding the value of goods bought or sold is similar to the method of finding the base when a part of it is given (Third Type Problem, Case 1, p. 92).

143. To find the Value of Goods bought or sold :

RULE. Divide the commission or brokerage by the rate per cent expressed as a decimal or common fraction.

EXERCISES

144. In each of the following, find the value of the goods bought or sold:

Commission	Rate of Commission	Commission	Rate of Commission
1. \$ 50	5 %	10. \$ 8	2 %
2. \$ 80	5 %	11. \$ 24	4 %
3. \$ 100	10 %	12. \$ 30	6 %
4. \$ 25	10 %	13. \$ 3	$1\frac{1}{2}$ %
5. \$ 7.50	1 %	14. \$ 15	$2\frac{1}{2}$ %
6. \$ 10	$\frac{1}{2}$ %	15. \$ 10	$\frac{1}{8}$ %
7. \$ 275	$\frac{2}{5}$ %	16. \$ 500	$1\frac{1}{4}$ %
8. \$ 480	$\frac{1}{4}$ %	17. \$ 7500	$\frac{1}{4}$ %
9. \$ 2500	$\frac{1}{8}$ %	18. \$ 1250	$\frac{1}{8}$ %

WRITTEN PROBLEMS

145. 1. An agent received \$200 for his commission for selling an automobile.

(a) Find the value of the automobile if the agent's commission was $12\frac{1}{2}$ %.

(b) How much did the firm receive for the automobile?

2. A real estate agent earned \$3000 in commissions in a year by selling property. Find the value of the property sold, if the agent's commissions were $1\frac{1}{2}$ % of the value of the property.

3. A commission merchant earned \$ 28 by selling apples at \$ 2 a barrel. His commission was 4 % of the value of the apples sold. How many barrels of apples did he sell?

4. An agent earned \$ 625 by selling pianos on commission, his rate of commission being 20 %.

(a) Find the value of the pianos sold.

(b) How many pianos did he sell, if each piano cost \$ 625?

5. A salesman is paid \$ 20 a week and 5 % commission for selling pictures. At the end of 2 weeks he received \$ 100. Find the value of the pictures sold.

146. To find the Amount to be Expended and the Commission, when Both are included in the Amount sent by the Principal to the Agent.

1. I send my agent \$ 630 with instructions to deduct a commission of 5 % and to buy apples with the balance. How much will be deducted as commission? How much will he invest in apples?

Process

\$ 630 includes money for apples and money for commission at 5 %.

Represent the money spent for apples by 100 %.

Represent the money spent for commission by 5 % of money spent for apples.

Then the amount sent to the agent equals 105 % of the amount spent for apples.

105 % of amount spent for apples = \$ 630.

Amount spent for apples = \$ 630 \div 1.05 = \$ 600.

\$ 630 - \$ 600 = \$ 30 Commission. *Ans.*

The method of solving this problem is similar to the method of finding the base when the base plus a part is given (Third Type Problem, Case 3, p. 100).

147. To find the Amount to be Expended for Goods, when the Amount sent to the Agent includes Both the Commission and the Money to be Expended.

RULE. Divide the total amount sent to the agent by 1 plus the rate of commission.

PROBLEMS

148. 1. Mr. Jones sent \$1947.50 to his agent with instructions to deduct his commission at $2\frac{1}{2}\%$ and to expend the remainder for produce.

(a) How much money is spent for the produce?

(b) How much does the agent keep for his commission?

2. An agent receives \$20,600 to pay for the cost of corn and for his commission at 3%.

(a) How much does he spend for the corn?

(b) How much does he deduct for his commission?

3. An agent receives \$1001.25 with instructions to deduct his commission at $\frac{7}{8}\%$, and to expend the remainder for wheat.

(a) How much money does he spend for wheat?

(b) How much does he deduct for his commission?

(c) How many bushels of wheat does he buy at \$1 per bushel?

4. An agent receives \$ 6597.50 to include the cost of flour and his commission at $1\frac{1}{2}\%$. He buys 1900 bbl. of flour.

- (a) How much money does he spend for flour?
- (b) How much does he pay per barrel?

TRADE DISCOUNT OR COMMERCIAL DISCOUNT

FOR READING AND DISCUSSION

149. Many manufacturers issue catalogues containing lists of the goods they sell and the prices at which the goods are sold. In these catalogues the goods are "listed" at a price greater than the firm expects to receive for them. The manufacturer then sends out discount lists, stating reductions or discounts from the catalogue or list prices.

For example, sewing machines may be listed at \$ 40 each, but the firm may allow a reduction or discount of 10% from the list price. The sewing machine, therefore, actually costs \$ 40 - \$ 4 or \$ 36.

150.

\$ 40 is called the **List Price** or **Catalogue Price**.

4 % is called the **Rate of Discount**.

\$ 4 is called the **Discount**.

\$ 36 is called the **Net Price** or **Actual Cost**.

Frequently two or more discounts are allowed. Automobiles may be listed at \$ 1500, with discounts of 20% and 5%. This means a discount of

20 % on \$ 1500, or \$300, making the price \$ 1200; and a further discount of 5 % on \$ 1200, or \$ 60, making the net cost of the automobiles \$ 1140.

Discounts are also given for payment before a bill is due. Usually a man who buys goods from one with whom he deals frequently, is allowed 30 da. in which to make payment. If he pays cash, he may be allowed a discount of 5 % or 2 %. Firms that buy large quantities of goods save considerable money each year by paying their bills promptly; they also obtain an excellent reputation among those with whom they deal.

151. To find the Discount and the Net Price, when the List Price and the Rates of Discount are Given

ONE DISCOUNT

1. The list price of a piano is \$ 650. The discount is 10 %. Find the discount and the net cost.

Process

$$\frac{1}{10} \text{ of } \$ 650 = \$ 65 \text{ Discount}$$

$$\$ 650 - \$ 65 = \$ 585 \text{ Net Cost}$$

Or

$$\frac{9}{10} \text{ of } \$ 650 = \$ 585 \text{ Net Cost}$$

TWO OR MORE DISCOUNTS

2. The list price of a piano is \$ 700. Discounts of 10 % and 2 % are allowed. Find the total discount and the net cost of the piano.

Process

$$\frac{1}{10} \text{ of } \$ 700 = \$ 70 \text{ Discount at } 10 \%$$

$$\$ 700 - \$ 70 = \$ 630 \text{ Price after deduction of first discount}$$

$$.02 \text{ of } \$ 630 = \$ 12.60 \text{ Discount at } 2 \%$$

$$\$ 630 - \$ 12.60 = \$ 617.40 \text{ Net Cost}$$

$$\$ 700 - \$ 617.40 = \$ 82.60 \text{ Total Discount}$$

There is a short method shown in the following:

3. Find the net cost of a phonograph listed at \$150, with discounts of 20 % and 5 %.

Process

A discount of 20 % leaves a net cost of 80 %

A discount of 5 % leaves a net cost of 95 %

The net cost = $.80 \times .95 \times \$150 = \114 . *Ans.*

Or

$$\frac{4}{5} \times \frac{19}{20} \times \$150 = \$114. \quad \text{Ans.}$$

$$\$150 - \$114 = \$36 \text{ Discount.} \quad \text{Ans.}$$

The method of finding the discount is similar to the method of finding a per cent of a number (First Type Problem, p. 70).

The method of finding the net price when one discount is allowed is similar to the method of finding the difference (p. 81).

152. RULE. To find the Net Price when more than one Discount is Allowed:

Deduct the first discount from the list price and each subsequent discount from each successive remainder. The last remainder is the net price.

NOTE. The order in which discounts are subtracted does not affect the final result; *e.g.* Discounts of 10, 5, and 2 may be deducted as 10, 5, and 2; or 5, 2, and 10; or 2, 5, and 10, etc.

The terms of discount may be written: 90 days net, 3 % 10 days; or net/90, 3/30. The terms of credit are usually 30, 60, or 90 days. The rates of discount vary.

ORAL EXERCISES

153. In each of the following find (a) the discount, (b) the net cost.

List Price	Rates of Discount	List Price	Rates of Discount
1. \$ 500	20 and 10	6. \$ 2000	10 and 2
2. \$ 1000	10 and 10	7. \$ 10,000	10 and 1
3. \$ 100	10 and 5	8. \$ 6000	5 and 2
4. \$ 200	10 and 2	9. \$ 1500	6 and 2
5. \$ 400	5 and 2	10. \$ 2500	10 and 5

NOTE. 20 and 10 means 20% and 10%. See also Art. 68, p. 57.

WRITTEN EXERCISES

154. Find the net cost and the total discount:

List Price	Rates of Discount	List Price	Rates of Discount
1. \$ 1200	20 and 5	11. \$ 1500	25 and 2
2. \$ 500	20 and 10	12. \$ 2500	25 and 2
3. \$ 200	25 and 2	13. \$ 150	30 and 2
4. \$ 600	30 and 2	14. \$ 360.25	5 and 5
5. \$ 75	10 and 2	15. \$ 140.75	10 and 1
6. \$ 40	10 and 5	16. \$ 240	$12\frac{1}{2}$ and 2
7. \$ 85	5 and 2	17. \$ 975	$33\frac{1}{3}$ and 5
8. \$ 115	40 and 10	18. \$ 2160	$16\frac{2}{3}$ and 5
9. \$ 235	50 and 10	19. \$ 1580	10, 5, and 2
10. \$ 740	25 and 10	20. \$ 4260	10, 5, and 2

ORAL PROBLEMS

155. 1. A storekeeper buys \$5000 worth of goods in a month. He is allowed 2 % off for paying cash. How much is saved by paying cash ?

2. A storekeeper buys \$2500 worth of furniture. He is allowed 2 % off for cash or 1 % off if he pays in 30 days. How much is saved by paying cash ?

3. A mahogany table is listed at \$125, with a discount of 20 %. Find the net cost.

4. A set of books containing 12 vols. is listed at \$5 a volume. Find the net cost of the set, if a discount of $33\frac{1}{3}$ % is allowed.

5. Mr. Gray buys 30 doz. knives and 30 doz. forks at \$9 a dozen. Find the net cost if he receives a discount of 10 %.

WRITTEN PROBLEMS

156. Find the net cost of the following articles :

1. 6 chairs at \$2.50 each ; discount of 25 %.

2. 30 books at \$3.50 each ; discount of 40 %.

3. 40 doz. fountain pens at \$1.25 each ; discounts of 30 % and 10 %.

4. 120 bbl. of flour at \$5.75 each ; discounts of 20 % and 5 %.

5. 72 crates of eggs, 6 doz. in a crate, at 25 ct. a dozen ; discount, 10 %.

Find the net cost and the discount in each of the following :

6. William Furman bought of George Burns :

6 doz. books, Style A, @ \$ 6.60 per dozen.

12 doz. books, Style B, @ \$ 8.40 per dozen.

18 doz. books, Style C, @ \$ 9.60 per dozen.

Discounts 40 % and 10 %.

7. John Davis bought of William Chandler :

5 M envelopes, No. 5, @ \$ 1.30 per M.

10 M envelopes, No. 6, @ \$ 2.20 per M.

50 M envelopes, No. 7, @ \$ 3.60 per M.

Discounts 5 % and 2 %.

8. Henry Henderson bought of George Wharton :

24 doz. lead pencils, American, @ \$ 4.38 a dozen.

36 doz. lead pencils, Eagle, @ \$ 4.37 a dozen.

60 doz. lead pencils, Dixon, @ \$ 4.69 a dozen.

Discounts $1\frac{1}{2}$ % and 1 %.

9. Henry Roth bought of Harry Tower :

10 gross pens, No. 10, @ \$.60 per gross.

20 gross pens, No. 20, @ \$.75 per gross.

25 gross pens, No. 30, @ \$ 1.10 per gross.

Discounts $2\frac{1}{2}$ % and 2 %.

10. Make out the bill, in proper form, for problem number 6. Find the net cost, allowing an additional discount of 2 % for cash.

11. Discounts of 20 % and 5 % were allowed from the list price of a canoe. If the canoe was listed at \$ 250, find the net cost.

12. How much do I pay for 6 rugs listed at \$ 150 each, if I am allowed discounts of 10 % and 5 % ?

13. I can buy goods listed at \$ 2000 at a discount of 25 %, or at a discount of 20 % and 5 % .

(a) Which rate gives me the greater discount ?

(b) How much greater ?

14. How much greater is a 30 % discount on \$ 2700 than a discount of 20 % and 10 % ?

15. (a) How much greater is a discount of 10 % and 5 % than a discount of 12 % ?

(b) How much less is a discount of 20 % and 10 % than a discount of 30 % ?

16. A large store buys \$10,000 worth of goods each month. If it pays cash, it receives 2 % discount. How much is saved in a year by paying cash ?

PROFIT AND LOSS

FOR READING AND DISCUSSION

157. Mr. Clark keeps a grocery store. He buys fruits, vegetables, cereals, etc., from the wholesale dealers and sells them to his customers. To carry on his business, he has to pay \$60 a month to a clerk, \$50 a month for rent, and \$5 for light. If Mr. Clark sold his goods at the prices he paid for them, he would not be able to pay these expenses nor make a profit for himself. Therefore, it is necessary to sell the goods at prices greater than the cost.

The price that Mr. Clark pays for his goods is the **Cost**; the price at which he sells them is the **Selling Price**. A **gain** or **profit** is made when the Selling Price is **GREATER** than the Cost; a **loss** is sustained when the Selling Price is **LESS** than the Cost. The per cent of the cost gained or lost is called the **per cent gained** or the **per cent lost**; it is also called the **rate of gain** or the **rate of loss**.

To find the Gain or Loss and the Selling Price when the Cost and the Rate of Gain or Loss are Given

158. 1. A book that cost \$2 is sold at a gain of 25%. Find the gain. Find the selling price.
2. A chair cost \$80. As it was damaged, it was sold at a loss of 20%. Find the loss. Find the selling price.

Process

Gain = 25%, or $\frac{1}{4}$ of the Cost.

Gain = $\frac{1}{4}$ of \$2 = \$.50. *Ans.*

Selling Price = Cost + Gain.

Selling Price = \$2 + \$.50.

Selling Price = \$2.50. *Ans.*

SECOND METHOD

Selling Price = $\frac{4}{4} + \frac{1}{4} = \frac{5}{4}$
of Cost.

Selling Price = $\frac{5}{4}$ of \$2.00.

Selling Price = \$2.50. *Ans.*

Process

Loss = 20% or $\frac{1}{5}$ of the Cost.

Loss = $\frac{1}{5}$ of \$80 = \$16. *Ans.*

Selling Price = Cost - Loss.

Selling Price = \$80 - \$16
= \$64. *Ans.*

SECOND METHOD

Selling Price = $\frac{5}{5} - \frac{1}{5} = \frac{4}{5}$
of Cost.

Selling Price = $\frac{4}{5}$ of \$80

= \$64. *Ans.*

The method of finding the **Gain** or **Loss** is similar to the method of finding the **Percentage** (First Type Problem, p. 70).

The method of finding the **Selling Price** (when there is a gain) is similar to the method of finding the **Amount** (p. 79).

159. The method of finding the **Selling Price** (when there is a loss) is similar to the method of finding the **Difference** (p. 81).

160. To find the Gain or Loss :

RULE. Multiply the cost by the rate of gain or loss expressed as a common fraction or a decimal.

FORMULA AND EQUATION. Gain or Loss = Rate \times Cost.

$$G \text{ or } L = R \times C$$

To find the Selling Price :

RULE. Add the gain to the cost or subtract the loss from the cost.

EQUATION AND FORMULA.

Selling Price = Cost + Gain, or Selling Price = Cost - Loss.

$$SP = C + G \text{ or } C - L.$$

ORAL PROBLEMS

161. 1. Pencils cost 2 ct. each. They are sold at a gain of 100 %. Find the gain. Find the selling price.

2. A blank book costs 4 ct. It is sold at a gain of 50 %. Find the gain.

3. Apples cost 3 ct. They are sold at a gain of $33\frac{1}{3}\%$. Find the gain on 1 doz. apples.

4. Butter which cost 30 ct. a pound is sold at a gain of $16\frac{2}{3}\%$.

- (a) Find the selling price of 1 lb.
- (b) Find the selling price of 6 lb.
- 5. Lead pencils that cost 5 ct. each are sold at a gain of 20 %. Find the selling price of 6 lead pencils.
- 6. Mr. Brown bought picture post cards at 3 ct. each and sold them at a gain of $66\frac{2}{3}$ %. How much did he receive for each card ?
- 7. Oranges that cost 8 ct. each are sold at a loss of 25 %. Find the loss. Find the selling price.
- 8. Books that cost \$ 1.50 each are sold at $33\frac{1}{3}$ % less than the cost. Find the selling price.
- 9. I bought 8 yd. of lace at 50 ct. a yard and sold it at a gain of 10 %. How much did I receive for the lace ?
- 10. A box of fruit that cost \$ 4 was sold at a loss of $12\frac{1}{2}$ %. Find the selling price.
- 11. Shoes that cost \$ 3 per pair are sold at a gain of 50 %. Find the selling price of 6 pr.
- 12. It costs a manufacturer \$ 1.50 to manufacture a cap. For how much must the cap be sold to gain 20 % ?
- 13. Bottles of ink cost 4 ct. each. For how much must each bottle be sold to gain 25 % ?
- 14. Baseballs that cost 50 ct. each are to be sold so as to gain 30 %. For how much must each ball be sold ?
- 15. One dozen neckties cost \$ 6. Find the selling price of each tie if I gain 20 %.

EXERCISES

162. Find the value of ? in each of the following:

	COST	GAIN PER CENT	LOSS PER CENT	GAIN	LOSS	SELLING PRICE
1.	\$ 3.00	50	—	?	—	?
2.	.15	100	—	?	—	?
3.	16.00	—	$12\frac{1}{2}$	—	?	?
4.	8.00	—	25	—	?	?
5.	200.00	20	—	?	—	?
6.	30.00	30	—	?	—	?
7.	250.00	—	50	—	?	?
8.	40.00	—	75	—	?	?
9.	90.00	$66\frac{2}{3}$	—	?	—	?
10.	17.00	10	—	?	—	?
11.	60.00	$16\frac{2}{3}$	—	?	—	?
12.	360.00	$33\frac{1}{3}$	—	?	—	?
13.	105.00	—	10	—	?	?
14.	80.00	40	—	?	—	?
15.	14.00	25	—	?	—	?

WRITTEN PROBLEMS

163. 1. A desk that cost \$55 was sold at a gain of 14%.

(a) Find the gain.

(b) Find the selling price.

2. How much is gained by selling a house that cost \$30,000 at a gain of 15%? How much is received for the house?

3. \$1500 worth of goods were sold at a loss of 12%.

(a) Find the loss.

(b) Find the selling price.

Find the gain and the selling price in each of the following:

4. A piano that cost \$500 was sold at a gain of 12%.

5. An automobile that cost \$1800 was sold at a gain of 21%.

6. A flag that cost \$45 was sold at a gain of 16%.

Find the loss and the selling price in each of the following:

7. A crate of oranges that cost \$12 was sold at a loss of 36%.

8. A boat that cost \$70 was sold at a loss of 23%.

9. A picture that cost \$240 was sold at a loss of 35%.

10. I bought two farms for \$10,000 each. I sold one of them at a gain of 18%, and the other at a loss of 18%. Did I gain or lose in all?

11. A grocer bought 40 boxes of fruit at \$2.50 per box. He sold it at a gain of 33%. How much did he receive for all the fruit?

12. 150 yd. of dress goods were bought at \$ 1.75 a yard and sold at an average profit of 40 %. How much was gained by the transaction ?

13. A newsboy bought 160 papers at 2 for 1 ct. He sold them at a gain of 100 %, and spent all the money for more papers at 2 for 1 ct. How many papers did he buy the second time ?

14. A farmer sells turkeys to an agent at 16 ct. a pound. The agent sells them to a butcher at an advance of 25 %. The butcher sells to a customer at a gain of 25 %.

(a) How much does the customer pay per pound ?

(b) How much more does the butcher receive per pound than the farmer ?

15. A stationer buys post cards at 2 for 5 ct. and sells them at a gain of 20 %. How much does he gain on 480 cards ?

16. If the stationer buys cards at the rate of 3 for 5 ct. and sells them at a gain of 30 %, how much does he receive for 50 doz. cards ?

17. 100 doz. drinking glasses are bought by a dealer at \$ 3 per doz. 10 % of them are broken. The remainder are sold for 20 % more than the cost. How much does the dealer gain or lose on the transaction ?

18. 75 lamp chimneys are bought for \$7.50. They are sold at a gain of 50 %. How much is received for each chimney ?

19. A contractor builds a fence. He employs 3 carpenters at \$4.50 per day. It takes 5 da. to build the fence. The lumber costs \$18. How much should the contractor charge in order to pay all his expenses and make a profit of 15 % for himself?

164. To find the Per Cent of Gain or Loss when the Selling Price and the Cost are Given

1. A bottle of ink that cost 4 ct. is sold at a gain of 1 ct. Find the per cent gain (the rate of gain).

Process

The gain is always calculated on the cost.

Therefore, the per cent gain = $\frac{1}{4}$, or 25 %. *Ans.*

2. A barrel of apples that cost \$2.50 is sold at a loss of 75 ct. Find the loss per cent (the rate of loss).

Process

The loss is always calculated on the cost.

Therefore, the per cent loss = $\frac{75}{250} = \frac{3}{10} = 30$ %. *Ans.*

From these problems we see that the method of finding the rate of gain or loss is similar to the method of finding what per cent one number is of another (Second Type Problem, p. 85).

165. $\left\{ \begin{array}{c} \text{The Rate of Gain} \\ \text{or} \\ \text{Gain Per Cent} \end{array} \right\}$ equals the Gain divided by the Cost.

$\left\{ \begin{array}{c} \text{The Rate of Loss} \\ \text{or} \\ \text{Loss Per Cent} \end{array} \right\}$ equals the Loss divided by the Cost.

EQUATION AND FORMULA.

$$\text{Rate of Gain} = \frac{G}{C}. \quad \text{Rate of Loss} = \frac{L}{C}.$$

PROBLEMS

166. 1. Tea that cost 40 ct. a pound is sold at a gain of 10 ct. per pound. Find the per cent gain.

2. A baseball that cost 25 ct. is sold at a gain of 10 ct. Find the per cent gain.

3. What per cent is gained by selling hats that cost \$ 2 at a gain of 50 ct. ?

4. Turkeys that cost 25 ct. a pound are sold at a loss of 3 ct. a pound. Find the per cent loss.

5. What per cent is lost by selling a bicycle that cost \$ 50 at a loss of \$ 25 ?

* 6. Coffee that cost 20 ct. a pound is sold for 25 ct. a pound. Find the per cent gain.

* 7. A box of fruit that cost \$ 6 is sold for \$ 4. Find the per cent loss.

† 8. 5 books are bought at 50 ct. each and sold at an advance of 25 ct. Find the per cent gain.

† 9. 2 books are bought at 50 ct. each and sold at a gain of 25 ct. each. Find the per cent gain.

† 10. Flour that cost 6 ct. a pound is sold for 8 ct. a pound. Find the gain per cent on 1 lb.

† 11. Flour that cost 5 ct. a pound is sold for 7 ct. a pound. Find the gain per cent on 10 lb.

12. 10 books are bought for \$ 20. They are sold at \$ 3 each. Find the per cent gain.

* NOTE. When the selling price and cost are given, find the gain or loss; then proceed as in numbers 1 and 2, p. 134.

† NOTE. Compare 8 and 9; 10 and 11.

WRITTEN PROBLEMS

166. 1. What per cent is gained by manufacturing hats at \$ 1.25 each and selling them at a gain of \$ 3 ?

2. What per cent is gained by buying pencils at 55 ct. a dozen and selling them at 72 ct. a dozen?

3. Library cards that cost 80 ct. per M. are sold at \$ 1.25 per M. Find the gain per cent.

4. What per cent is lost by selling books that cost \$ 2.75 each at a loss of 75 ct. each ?

5. What per cent is lost by buying shirts at \$ 1.50 each and selling them at \$ 1.30 each ?

6. Flowers that cost 90 ct. a hundred are sold at 78 ct. a hundred. Find the loss per cent.

7. Shoes are bought for \$ 1.80 per pair and sold for \$ 2.25 per pair. Find the gain per cent on 1 pr.

8. Shoes are bought for \$ 2.25 per pair and sold for \$ 2.60 per pair.

(a) Find the gain per cent on 1 pr.

(b) Find the gain per cent on 100 pr.

9. If a grocer makes a profit of 25% on each pound of flour he sells, what per cent profit does he make by selling a barrel of flour ?

10. Penholders are bought at 35 ct. a dozen and sold at 5 ct. each. Find the per cent gain.

11. Flour is bought at \$ 6 a barrel and sold at the rate of $5\frac{1}{2}$ ct. a pound. Find the per cent gain.

12. Post cards are bought at 50 ct. a hundred and sold at the rate of 3 for 5 ct. Find the gain per cent.

13. A can of milk containing 40 qt. costs \$ 1.60. What per cent is gained by selling the milk for 6 ct. a quart?

14. A florist bought 72 roses for 80 ct. He made bouquets of them, putting 12 roses into each bouquet. What per cent profit did he make if he sold the bouquets for 25 ct. each?

15. A retail coal dealer buys a ton of coal for \$ 5.50. He sells the coal by the pailful, charging 25 ct. for each pailful. What per cent profit does he make if he gets 40 pailfuls from each ton?

16. An iceman pays 30 ct. for each 100 lb. of ice. He sells it in 10-ct. pieces. What per cent profit does he make if he cuts 7 pieces out of each 100 lb.?

167. To find the Cost when the Gain and the Gain Per Cent are Given

Or

To find the Cost when the Loss and the Loss Per Cent are Given

WHEN THE RATE IS REDUCIBLE TO A COMMON FRACTION

168. 1. I sold a book at a gain of 60 ct., thereby gaining $37\frac{1}{2}\%$ of the cost. Find the cost.

Process**FIRST METHOD**

$$37\frac{1}{2}\% = \frac{3}{8}.$$

$$\frac{3}{8} \text{ of the Cost} = 60 \text{ ct.}$$

$$\frac{1}{8} \text{ of the Cost} = \frac{1}{8} \text{ of } 60 \text{ ct.} \\ = 20 \text{ ct.}$$

$$\frac{3}{8} \text{ or the Cost} = 8 \times 20 \text{ ct.} \\ = \$1.60 \text{ Cost.}$$

*Ans.***OR SHORTER METHOD**

$$\frac{3}{8} \text{ of the Cost} = 60 \text{ ct.}$$

$$\text{Cost} = 60 \text{ ct.} \div \frac{3}{8}$$

$$\text{Cost} = 60 \text{ ct.} \times \frac{8}{3} \\ = \$1.60 \text{ Cost.}$$

*Ans.***WHEN THE RATE IS REDUCIBLE TO A DECIMAL FRACTION**

2. I sold a horse at a loss of \$80, thereby losing 16%. Find the cost.

Process

$$16\% \text{ of the Cost} = \$80.$$

$$\text{Cost} = \$80 \div .16 = \$500.$$

Ans.

From these problems, we see that the method of finding the **Cost** is similar to the method of finding the **Base** when the percentage and the rate per cent are given (Third Type Problem, Case 1, p. 93).

169. To find the Cost when the Gain and Gain Per Cent or Loss and Loss Per Cent are Given:

RULE. Divide the gain or loss by the gain or loss per cent.

EQUATION AND FORMULA.

$$\text{Cost} = \text{Gain} \div \text{Gain Per Cent} \quad \left(C = \frac{G}{G\%} \right).$$

$$\text{Cost} = \text{Loss} \div \text{Loss Per Cent} \quad \left(C = \frac{L}{L\%} \right).$$

ORAL PROBLEMS

170. 1. By selling meat at a gain of 4 ct. a pound a butcher makes a profit of 25%. Find the cost of 1 lb.

2. Flour is sold at a gain of 50 %. If the actual gain is $2\frac{1}{2}$ ct. per pound, find the cost of 1 lb. of flour.

3. I gain $66\frac{2}{3}\%$, or 2 ct., on each lead pencil sold. How much do I pay for each pencil?

4. A bicycle was sold at a gain of \$5. This was 10 % of the cost. Find the cost.

5. Eggs were sold at a loss of \$9. How much did they cost if the loss was $37\frac{1}{2}\%$ of the cost?

6. A real estate dealer made a profit of \$500 on a plot of ground. The rate of profit was 20 %. Find the cost of the plot.

7. Butter was sold at a loss of 9 ct., or 30 %, on each pound. Find the cost of 1 lb.

8. A profit of 24 ct., or 40 %, was made on a picture. Find the cost of the picture.

9. Find the cost of a boat, if a loss of \$9 equals $12\frac{1}{2}\%$ of the cost.

10. Shirts were sold at a profit of \$3 a dozen, thereby gaining 50 %. How much did 6 doz. shirts cost?

WRITTEN PROBLEMS

171. 1. A coal dealer makes a profit of 77 ct., or 14 %, on each ton of coal he sells. Find how much he paid for each ton.

2. A barrel of potatoes was sold at a profit of 35 ct., or $23\frac{1}{3}\%$. Find the cost.

3. If I gain 38 %, or \$199.50, by selling a piano, how much did I pay for it ?

4. A house and lot were sold at a profit of \$3410. The rate of profit was 22 %.

(a) Find the cost of the house and lot.

(b) Find the selling price.

5. A profit of \$1710, or 18 %, was made on an automobile. Find the cost and the selling price.

6. By selling a phonograph at a gain of \$23.25, a dealer made a profit of 31 %. Find cost of the phonograph.

7. By selling a suit of clothes at a loss of 8 %, a tailor lost \$142. Find the selling price.

8. Goods that had been damaged by fire were sold at a loss of 15 %. The actual loss was \$187.50. Find the cost and the selling price.

9. A gain of 250 % was made by selling a diamond ring. The actual gain was \$150. Find the cost.

10. A dry goods store sells its goods at a profit of 18 %. Find the cost of a hat on which a profit of \$1.44 is made. Find the selling price.

11. An agent sold 8 doz. books. He made a profit of 40 % on each book. Find the total cost of the books, if the profit on each book was \$1.20.

172. To find the Cost when the Selling Price and the Gain Per Cent are Given

WHEN THE RATE IS REDUCED
TO A COMMON FRACTION

1. A desk was sold for \$60.
This was at a gain of 25%.
Find the cost.

Process

Gain = $\frac{1}{4}$ of Cost.
Represent the Cost by $\frac{4}{4}$.
Selling Price = $\frac{5}{4}$ of Cost.
Therefore, $\frac{5}{4}$ of Cost = \$60.
Cost = $\$60 \div \frac{5}{4}$
= $\$60 \times \frac{4}{5}$ = \$48. *Ans.*

WHEN THE RATE IS REDUCED
TO A DECIMAL FRACTION

2. By selling a picture for
\$168, I gained 12%. How
much did the picture cost?

Process

Gain = 12% of the Cost.
Represent the Cost by 100%.
Selling Price = 112% of the
Cost.
Therefore, 112% of the Cost
= \$168.
Cost = $\$168 \div 1.12$
= \$150. *Ans.*

From problems 1 and 2 we see that the method of finding the **Cost** when the **Selling Price** and the **Gain Per Cent** are given is similar to the method of finding the **Base** when the **Amount** and **Rate of Increase** are given (Third Type Problem, Case 3, p. 100).

173. To find the Cost when the Selling Price and the Gain Per Cent are Given:

RULE. Divide the selling price by 1 plus the gain per cent.

EQUATION AND FORMULA.

$$C = SP \div (1 + G\%), \text{ or } C = \frac{SP}{1 + G\%}.$$

174. To find the Cost when the Selling Price and the Loss Per Cent are Given

WHEN THE RATE IS REDUCED
TO A COMMON FRACTION

1. A plot of ground was sold at a loss of $16\frac{2}{3}\%$. The plot was sold for \$2000. Find the cost.

Process

The Loss = $\frac{1}{6}$ of the Cost.
Represent the Cost by $\frac{x}{6}$, and
the S. P. = $\frac{5}{6}$ of the Cost.

Therefore, $\frac{5}{6}$ of the Cost =
\$2000.

Cost = \$2000 $\div \frac{5}{6}$ or
 $2000 \times \frac{6}{5} = \$2400.$ *Ans.*

WHEN THE RATE IS REDUCED
TO A DECIMAL FRACTION

2. A carriage was sold for \$322, which was at a loss of 8%. How much did the carriage cost?

Process

The Loss = 8% of the Cost.
Represent the Cost by 100%,
the S. P. = 92% of the Cost.

Therefore, 92% of the cost
= \$322.

Cost = \$322 $\div .92 = \$350.$
Ans.

From problems 1 and 2 we see that the method of finding the **Cost** when the **Selling Price** and the **Loss Per Cent** are given is similar to the method of finding the **Base** when the **Difference** and the **Rate of Decrease** are given (Third Type Problem, Case 2, p. 98).

175. To find the Cost when the Selling Price and the Loss Per Cent are Given:

RULE. Divide the selling price by 1 minus the loss per cent.

EQUATION AND FORMULA.

$$C = SP \div (1 - L\%), \text{ or } C = \frac{SP}{1 - L\%}.$$

ORAL EXERCISES

176. Find the cost of the following:

Selling Price	Gain Per Cent	Selling Price	Gain Per Cent
1. \$ 100	20	7. \$ 200	25
2. \$ 90	50	8. \$ 36	$12\frac{1}{2}$
3. \$ 40	$33\frac{1}{3}$	9. \$100	100
4. \$ 90	25	10. \$ 60	$33\frac{1}{3}$
5. \$ 50	50	11. \$ 80	20
6. \$ 45	10	12. \$ 75	$66\frac{2}{3}$

WRITTEN EXERCISES

177. Find the cost in each of the following:

Selling Price	Gain or Loss	Selling Price	Gain or Loss
1. \$ 232.50	L., 7%	6. \$ 226.00	G., 13%
2. \$ 220.40	L., 24%	7. \$ 378.00	L., 16%
3. \$ 940.00	L., 6%	8. \$ 682.50	L., 9%
4. \$ 1470.00	L., 2%	9. \$ 1300.00	G., 100%
5. \$ 262.60	G., 30%	10. \$ 500.00	L., 20%

WRITTEN EXERCISES

178. Find the cost in each of the following:

Selling Price	Gain or Loss	Selling Price	Gain or Loss
1. \$ 146.25	L., $21\frac{1}{2}$ %	6. \$ 71.82	L., $51\frac{1}{2}$ %
2. \$ 309.75	G., $31\frac{1}{4}$ %	7. \$ 668.00	L., $16\frac{1}{2}$ %
3. \$ 2610.00	G., $30\frac{1}{2}$ %	8. \$ 259.80	G., $8\frac{1}{4}$ %
4. \$ 626.50	L., $10\frac{1}{2}$ %	9. \$ 935.75	L., $11\frac{1}{2}$ %
5. \$ 1518.75	G., $11\frac{1}{4}$ %	10. \$ 2412.50	L., $31\frac{1}{2}$ %

19. A contractor builds a fence. He employs 3 carpenters at \$4.50 per day. It takes 5 da. to build the fence. The lumber costs \$18. How much should the contractor charge in order to pay all his expenses and make a profit of 15 % for himself?

164. To find the Per Cent of Gain or Loss when the Selling Price and the Cost are Given

1. A bottle of ink that cost 4 ct. is sold at a gain of 1 ct. Find the per cent gain (the rate of gain).

Process

The gain is always calculated on the cost.

Therefore, the per cent gain = $\frac{1}{4}$, or 25 %. *Ans.*

2. A barrel of apples that cost \$2.50 is sold at a loss of 75 ct. Find the loss per cent (the rate of loss).

Process

The loss is always calculated on the cost.

Therefore, the percent loss = $\frac{75}{250} = \frac{3}{10} = 30$ %. *Ans.*

From these problems we see that the method of finding the rate of gain or loss is similar to the method of finding what per cent one number is of another (Second Type Problem, p. 85).

165. RULE. $\left\{ \begin{array}{c} \text{The Rate of Gain} \\ \text{or} \\ \text{Gain Per Cent} \end{array} \right\}$ equals the Gain divided by the Cost.

$\left\{ \begin{array}{c} \text{The Rate of Loss} \\ \text{or} \\ \text{Loss Per Cent} \end{array} \right\}$ equals the Loss divided by the Cost.

EQUATION AND FORMULA.

$$\text{Rate of Gain} = \frac{G}{C}. \quad \text{Rate of Loss} = \frac{L}{C}.$$

PROBLEMS

166. 1. Tea that cost 40 ct. a pound is sold at a gain of 10 ct. per pound. Find the per cent gain.

2. A baseball that cost 25 ct. is sold at a gain of 10 ct. Find the per cent gain.

3. What per cent is gained by selling hats that cost \$ 2 at a gain of 50 ct. ?

4. Turkeys that cost 25 ct. a pound are sold at a loss of 3 ct. a pound. Find the per cent loss.

5. What per cent is lost by selling a bicycle that cost \$ 50 at a loss of \$ 25 ?

* 6. Coffee that cost 20 ct. a pound is sold for 25 ct. a pound. Find the per cent gain.

* 7. A box of fruit that cost \$ 6 is sold for \$ 4. Find the per cent loss.

† 8. 5 books are bought at 50 ct. each and sold at an advance of 25 ct. Find the per cent gain.

† 9. 2 books are bought at 50 ct. each and sold at a gain of 25 ct. each. Find the per cent gain.

† 10. Flour that cost 6 ct. a pound is sold for 8 ct. a pound. Find the gain per cent on 1 lb.

† 11. Flour that cost 5 ct. a pound is sold for 7 ct. a pound. Find the gain per cent on 10 lb.

12. 10 books are bought for \$ 20. They are sold at \$ 3 each. Find the per cent gain.

* NOTE. When the selling price and cost are given, find the gain or loss ; then proceed as in numbers 1 and 2, p. 134.

† NOTE. Compare 8 and 9 ; 10 and 11.

WRITTEN PROBLEMS

166. 1. What per cent is gained by manufacturing hats at \$ 1.25 each and selling them at a gain of \$ 3 ?

2. What per cent is gained by buying pencils at 55 ct. a dozen and selling them at 72 ct. a dozen?

3. Library cards that cost 80 ct. per M. are sold at \$ 1.25 per M. Find the gain per cent.

4. What per cent is lost by selling books that cost \$ 2.75 each at a loss of 75 ct. each ?

5. What per cent is lost by buying shirts at \$ 1.50 each and selling them at \$ 1.30 each ?

6. Flowers that cost 90 ct. a hundred are sold at 78 ct. a hundred. Find the loss per cent.

7. Shoes are bought for \$ 1.80 per pair and sold for \$ 2.25 per pair. Find the gain per cent on 1 pr.

8. Shoes are bought for \$ 2.25 per pair and sold for \$ 2.60 per pair.

(a) Find the gain per cent on 1 pr.

(b) Find the gain per cent on 100 pr.

9. If a grocer makes a profit of 25% on each pound of flour he sells, what per cent profit does he make by selling a barrel of flour ?

10. Penholders are bought at 35 ct. a dozen and sold at 5 ct. each. Find the per cent gain.

11. Flour is bought at \$ 6 a barrel and sold at the rate of $5\frac{1}{2}$ ct. a pound. Find the per cent gain.

12. Post cards are bought at 50 ct. a hundred and sold at the rate of 3 for 5 ct. Find the gain per cent.

13. A can of milk containing 40 qt. costs \$ 1.60. What per cent is gained by selling the milk for 6 ct. a quart?

14. A florist bought 72 roses for 80 ct. He made bouquets of them, putting 12 roses into each bouquet. What per cent profit did he make if he sold the bouquets for 25 ct. each?

15. A retail coal dealer buys a ton of coal for \$ 5.50. He sells the coal by the pailful, charging 25 ct. for each pailful. What per cent profit does he make if he gets 40 pailfuls from each ton?

16. An iceman pays 30 ct. for each 100 lb. of ice. He sells it in 10-ct. pieces. What per cent profit does he make if he cuts 7 pieces out of each 100 lb.?

167. To find the Cost when the Gain and the Gain Per Cent are Given

Or

To find the Cost when the Loss and the Loss Per Cent are Given

WHEN THE RATE IS REDUCIBLE TO A COMMON FRACTION

168. 1. I sold a book at a gain of 60 ct., thereby gaining $37\frac{1}{2}\%$ of the cost. Find the cost.

Process**FIRST METHOD****OR SHORTER METHOD**

$37\frac{1}{2}\% = \frac{3}{8}$.	$\frac{3}{8}$ of the Cost = 60 ct.
$\frac{3}{8}$ of the Cost = 60 ct.	Cost = 60 ct. $\div \frac{3}{8}$.
$\frac{1}{8}$ of the Cost = $\frac{1}{8}$ of 60 ct.	Cost = 60 ct. $\times \frac{8}{3}$
= 20 ct.	= \$1.60 Cost.
$\frac{3}{8}$ or the Cost = 8×20 ct.	<i>Ans.</i>
= \$1.60 Cost.	
<i>Ans.</i>	

WHEN THE RATE IS REDUCIBLE TO A DECIMAL FRACTION

2. I sold a horse at a	Process
loss of \$ 80, thereby los-	16 % of the Cost = \$ 80.
ing 16 %. Find the cost.	Cost = \$ 80 \div .16 = \$ 500.
	<i>Ans.</i>

From these problems, we see that the method of finding the **Cost** is similar to the method of finding the **Base** when the percentage and the rate per cent are given (Third Type Problem, Case 1, p. 93).

169. To find the Cost when the Gain and Gain Per Cent or Loss and Loss Per Cent are Given:

RULE. Divide the gain or loss by the gain or loss per cent.

EQUATION AND FORMULA.

$$\text{Cost} = \text{Gain} \div \text{Gain Per Cent} \quad \left(C = \frac{G}{G\%} \right).$$

$$\text{Cost} = \text{Loss} \div \text{Loss Per Cent} \quad \left(C = \frac{L}{L\%} \right).$$

ORAL PROBLEMS

170. 1. By selling meat at a gain of 4 ct. a pound a butcher makes a profit of 25 %. Find the cost of 1 lb.

2. Flour is sold at a gain of 50 %. If the actual gain is $2\frac{1}{2}$ ct. per pound, find the cost of 1 lb. of flour.

3. I gain $66\frac{2}{3}\%$, or 2 ct., on each lead pencil sold. How much do I pay for each pencil?

4. A bicycle was sold at a gain of \$5. This was 10 % of the cost. Find the cost.

5. Eggs were sold at a loss of \$9. How much did they cost if the loss was $37\frac{1}{2}\%$ of the cost?

6. A real estate dealer made a profit of \$500 on a plot of ground. The rate of profit was 20 %. Find the cost of the plot.

7. Butter was sold at a loss of 9 ct., or 30 %, on each pound. Find the cost of 1 lb.

8. A profit of 24 ct., or 40 %, was made on a picture. Find the cost of the picture.

9. Find the cost of a boat, if a loss of \$9 equals $12\frac{1}{2}\%$ of the cost.

10. Shirts were sold at a profit of \$3 a dozen, thereby gaining 50 %. How much did 6 doz. shirts cost?

WRITTEN PROBLEMS

171. 1. A coal dealer makes a profit of 77 ct., or 14 %, on each ton of coal he sells. Find how much he paid for each ton.

2. A barrel of potatoes was sold at a profit of 35 ct., or $23\frac{1}{3}\%$. Find the cost.

3. If I gain 38 %, or \$ 199.50, by selling a piano, how much did I pay for it ?

4. A house and lot were sold at a profit of \$ 3410. The rate of profit was 22 %.

(a) Find the cost of the house and lot.

(b) Find the selling price.

5. A profit of \$ 1710, or 18 %, was made on an automobile. Find the cost and the selling price.

6. By selling a phonograph at a gain of \$ 23.25, a dealer made a profit of 31%. Find cost of the phonograph.

7. By selling a suit of clothes at a loss of 8%, a tailor lost \$142. Find the selling price.

8. Goods that had been damaged by fire were sold at a loss of 15%. The actual loss was \$187.50. Find the cost and the selling price.

9. A gain of 250% was made by selling a diamond ring. The actual gain was \$ 150. Find the cost.

10. A dry goods store sells its goods at a profit of 18 %. Find the cost of a hat on which a profit of \$ 1.44 is made. Find the selling price.

11. An agent sold 8 doz. books. He made a profit of 40 % on each book. Find the total cost of the books, if the profit on each book was \$ 1.20.

172. To find the Cost when the Selling Price and the Gain Per Cent are Given

WHEN THE RATE IS REDUCED
TO A COMMON FRACTION

1. A desk was sold for \$60. This was at a gain of 25%. Find the cost.

Process

Gain = $\frac{1}{4}$ of Cost.
Represent the Cost by $\frac{1}{4}$.
Selling Price = $\frac{5}{4}$ of Cost.
Therefore, $\frac{5}{4}$ of Cost = \$60.
Cost = $\$60 \div \frac{5}{4}$
= $\$60 \times \frac{4}{5} = \48 . *Ans.*

WHEN THE RATE IS REDUCED
TO A DECIMAL FRACTION

2. By selling a picture for \$168, I gained 12%. How much did the picture cost?

Process

Gain = 12% of the Cost.
Represent the Cost by 100%.
Selling Price = 112% of the Cost.
Therefore, 112% of the Cost = \$168.
Cost = $\$168 \div 1.12$
= \$150. *Ans.*

From problems 1 and 2 we see that the method of finding the **Cost** when the **Selling Price** and the **Gain Per Cent** are given is similar to the method of finding the **Base** when the **Amount** and **Rate of Increase** are given (Third Type Problem, Case 3, p. 100).

173. To find the Cost when the Selling Price and the Gain Per Cent are Given:

RULE. Divide the selling price by 1 plus the gain per cent.

EQUATION AND FORMULA.

$$C = SP \div (1 + G\%), \text{ or } C = \frac{SP}{1 + G\%}.$$

WRITTEN EXERCISES

Find the cost in each of the following:

179. 1. Selling Price, \$930.00 Gain, $3\frac{1}{3}\%$

Process

$1.03\frac{1}{3})\$930$ Multiply both divisor and dividend by 3.
 $3.10)\$2790 = \$900.$ *Ans.*

	Selling Price	Gain or Loss		Selling Price	Gain or Loss
2.	\$ 532.00	L., $6\frac{2}{3}\%$	3.	\$ 343.00	G., $16\frac{2}{3}\%$
4.	\$ 521.90	G., $2\frac{1}{3}\%$	5.	\$ 393.00	L., $12\frac{2}{3}\%$

ORAL PROBLEMS

180. 1. Oranges are sold at 24 ct. a dozen. If this is at a gain of 20 %, find the cost of 1 doz.

2. Magazines are sold at 10 ct. each. This is at a gain of 25 %. Find the cost.

3. By selling newspapers at 5 ct. each, a boy makes 100 %. How much did each newspaper cost?

4. A baseball bat was sold at a loss of $33\frac{1}{3}\%$. Find the cost of the bat, if the selling price was 40 ct.

5. A barrel of apples was sold for \$ 2.80, which was at a loss of $16\frac{2}{3}\%$. Find the cost.

6. Picture post cards are sold at a gain of 40%. If they are sold for 14 ct. a dozen, how much did they cost per dozen?

ORAL EXERCISES

176. Find the cost of the following:

Selling Price	Gain Per Cent	Selling Price	Gain Per Cent
1. \$ 100	20	7. \$ 200	25
2. \$ 90	50	8. \$ 36	$12\frac{1}{2}$
3. \$ 40	$33\frac{1}{3}$	9. \$ 100	100
4. \$ 90	25	10. \$ 60	$33\frac{1}{3}$
5. \$ 50	50	11. \$ 80	20
6. \$ 45	10	12. \$ 75	$66\frac{2}{3}$

WRITTEN EXERCISES

177. Find the cost in each of the following:

Selling Price	Gain or Loss	Selling Price	Gain or Loss
1. \$ 232.50	L., 7%	6. \$ 226.00	G., 13%
2. \$ 220.40	L., 24%	7. \$ 378.00	L., 16%
3. \$ 940.00	L., 6%	8. \$ 682.50	L., 9%
4. \$ 1470.00	L., 2%	9. \$ 1300.00	G., 100%
5. \$ 262.60	G., 30%	10. \$ 500.00	L., 20%

WRITTEN EXERCISES

178. Find the cost in each of the following:

Selling Price	Gain or Loss	Selling Price	Gain or Loss
1. \$ 146.25	L., $2\frac{1}{2}$ %	6. \$ 71.82	L., $5\frac{1}{2}$ %
2. \$ 309.75	G., $3\frac{1}{4}$ %	7. \$ 668.00	L., $16\frac{1}{2}$ %
3. \$ 2610.00	G., $30\frac{1}{2}$ %	8. \$ 259.80	G., $8\frac{1}{4}$ %
4. \$ 626.50	L., $10\frac{1}{2}$ %	9. \$ 935.75	L., $1\frac{1}{2}$ %
5. \$ 1518.75	G., $1\frac{1}{4}$ %	10. \$ 2412.50	L., $3\frac{1}{2}$ %

WRITTEN EXERCISES

Find the cost in each of the following:

179. 1. Selling Price, \$930.00 Gain, $31\frac{1}{3}\%$

Process

$1.034\overline{) \$930}$ Multiply both divisor and dividend by 3.
 $3.10\overline{) \$2790} = \$900.$ *Ans.*

	Selling Price	Gain or Loss		Selling Price	Gain or Loss
2.	\$ 532.00	L., $6\frac{2}{3}\%$	3.	\$ 343.00	G., $16\frac{2}{3}\%$
4.	\$ 521.90	G., $21\frac{1}{3}\%$	5.	\$ 393.00	L., $12\frac{2}{3}\%$

ORAL PROBLEMS

180. 1. Oranges are sold at 24 ct. a dozen. If this is at a gain of 20 %, find the cost of 1 doz.

2. Magazines are sold at 10 ct. each. This is at a gain of 25 %. Find the cost.

3. By selling newspapers at 5 ct. each, a boy makes 100 %. How much did each newspaper cost?

4. A baseball bat was sold at a loss of $33\frac{1}{3}\%$. Find the cost of the bat, if the selling price was 40 ct.

5. A barrel of apples was sold for \$ 2.80, which was at a loss of $16\frac{2}{3}\%$. Find the cost.

6. Picture post cards are sold at a gain of 40%. If they are sold for 14 ct. a dozen, how much did they cost per dozen?

(b) How much money was gained on each barrel?

12. I sold two farms for \$25,000. On one I gained 20%; on the other I lost 20%. How much did I gain or lose in all?

13. I bought two farms for \$20,000 each. I sold one at a gain of 17% and the other at a loss of 17%. How much did I gain or lose in all?

14. 12,500 bu. of wheat were purchased by Mr. Jones. He sold them for \$14,375, which was 15% more than he paid for them.

(a) How much did Mr. Jones pay for the wheat per bushel?

(b) How much did he receive for the wheat per bushel?

15. A profit of $33\frac{1}{3}\%$ was made by selling 80 cows for \$6000. How much did each cow cost?

16. Mr. Harris bought 10 plots of ground and sold them at an average profit of 18%.

(a) How much did he pay for all the plots, if he sold all of them for \$29,500?

(b) Find the gain on each plot.

17. 60 boxes of pencils, each box containing 6 doz. pencils, were sold at a gain of 30%. Find the cost of each pencil, if the total selling price was \$112.32.

3. Eggs are sold at \$ 6.08 per box. This was a loss of $33\frac{1}{3}\%$. Find the cost.

4. A farm was sold for \$ 4140, which was a gain of 15%. Find the cost.

5. By selling a farm of 20 A. for \$ 7000, Mr. Sheldon gained $14\frac{2}{7}\%$. How much did he pay for the farm per acre?

6. Cranberries were sold at a loss of 20%. How much did the berries cost, if they were sold for \$ 560?

7. 200 bbl. of cranberries were sold for \$ 1080, which was at a gain of 8%. Find the cost of the berries per barrel.

8. A load of melons was sold at $81\frac{1}{3}\%$ above cost. If the selling price was \$ 1300, what was the cost?

9. 350 crates of melons were sold at a gain of $16\frac{2}{3}\%$. If the selling price was \$ 525, what was the cost of the melons per crate?

10. 50 bbl. of cabbages were sold for \$ 32.20. This was 10 % less than the cost. How much did the cabbages cost?

11. 125 bbl. of cabbages were sold for \$ 93.75. This was 25 % greater than the cost.

How much did the cabbages cost per
1?

(b) How much money was gained on each barrel ?

12. I sold two farms for \$25,000. On one I gained 20 % ; on the other I lost 20 %. How much did I gain or lose in all ?

13. I bought two farms for \$20,000 each. I sold one at a gain of 17 % and the other at a loss of 17 %. How much did I gain or lose in all ?

14. 12,500 bu. of wheat were purchased by Mr. Jones. He sold them for \$14,375, which was 15 % more than he paid for them.

(a) How much did Mr. Jones pay for the wheat per bushel ?

(b) How much did he receive for the wheat per bushel ?

15. A profit of $33\frac{1}{3}$ % was made by selling 80 cows for \$6000. How much did each cow cost ?

16. Mr. Harris bought 10 plots of ground and sold them at an average profit of 18 %.

(a) How much did he pay for all the plots, if he sold all of them for \$29,500 ?

(b) Find the gain on each plot.

17. 60 boxes of pencils, each box containing 6 doz. pencils, were sold at a gain of 30 %. Find the cost of each pencil, if the total selling price was \$112.32.

EXERCISES

182. Find values for ? in each of the following.
(Use paper and pencil only when necessary.)

COST	SELLING PRICE	GAIN IN MONEY	LOSS IN MONEY	GAIN PER CENT	LOSS PER CENT
1. \$ 6	\$ 9	?	—	?	—
2. \$ 8	\$ 6	—	?	—	?
3. ?	\$ 10	\$ 5	—	?	—
4. ?	?	—	\$ 5	—	20 %
5. \$ 20	?	\$ 4	—	?	—
6. \$ 50	?	—	\$ 10	—	?
7. \$ 40	?	?	—	12½ %	—
8. \$ 10	?	?	—	50 %	—
9. \$ 60	?	—	?	—	33⅓ %
10. \$ 32	?	—	?	—	37½ %
11. ?	?	\$ 9	—	25 %	—
12. ?	?	—	\$ 12	—	20 %
13. \$ 7.50	\$ 15	?	—	?	—
14. \$ 12	\$ 6	—	?	—	?
15. ?	?	\$ 2.50	—	33⅓ %	—
16. ?	?	\$ 4.50	—	—	25 %
17. ?	\$ 50	?	—	25 %	—
18. ?	\$ 20	—	?	—	20 %
19. ?	\$ 16	?	—	33⅓ %	—
20. ?	\$ 12	—	?	—	25 %
21. ?	?	\$ 2.50	—	10 %	—
22. ?	?	—	\$ 5	—	12½ %
23. \$ 4	\$ 12	?	—	?	—
24. ?	\$ 15	—	?	—	16⅔ %
25. ?	\$ 30	?	—	66⅔ %	—

TAXES

FOR READING AND DISCUSSION

183. All governments spend money for schools, for police departments and fire departments, for prisons, for salaries, for roads, etc. The government obtains the money it needs by taxing the people.

Some of the ways in which most of the money is obtained are :

1. By levying a tax on the value of real estate. This tax is paid directly by each person who owns real estate; it is paid indirectly by each person who pays rent.

2. By levying a tax on the value of personal property, *e.g.* pictures, books, jewelry, securities, etc. This tax is paid directly by each person who owns personal property.

3. By levying a tax on incomes; this tax is called an **income tax**; it is paid by each person whose income exceeds a certain amount of money.

184. A tax rate of 2 % means that

For every \$ 1 of property, a person must pay 2 ct. in taxes.

For every \$ 100 of property, a person must pay \$ 2 in taxes.

For every \$ 1000 of property, a person must pay \$ 20 in taxes.

To find the Amount of Money to be paid in Taxes

To find the amount of money to be paid in taxes, proceed as in the First Type Problem in Percentage (p. 70).

185. The value of the property corresponds to the base. The tax rate corresponds to the rate per cent. The amount of taxes corresponds to the percentage.

ORAL EXERCISES

186. Find the amount of taxes paid by each of the following men who live in the town of Danville. The tax rate is $1\frac{1}{2}\%$.

NAME	VALUE OF PROPERTY	NAME	VALUE OF PROPERTY
1. Mr. Jordan	\$10,000	4. Mr. Dolan	\$24,000
2. Mr. Hurley	\$6,000	5. Mr. Gruber	\$30,000
3. Mr. Bates	\$18,000	6. Mr. Walsh	\$35,000

7. Find the taxes on Mr. Jordan's property, if it is taxed on $\frac{4}{5}$ of its value at $1\frac{3}{4}\%$ per annum.

8. Mr. Jones is taxed \$3 on every \$100 of property. How much does he pay on \$15,000 worth of property?

9. Find the amount Mr. Hayes pays in taxes on \$28,000 worth of property at $1\frac{1}{8}\%$.

10. If the rate is \$4 on \$100, find the amount paid by Mr. Cameron, whose property is worth \$40,000.

11. Find the taxes on \$20,000 worth of property at 2.15 %.

12. Find the taxes on \$30,000 worth of property at 1.54 %.

WRITTEN PROBLEMS

187. If the tax rate is 2.14 %, find the amount paid in each of the following :

NAME	VALUE OF PROPERTY
1. Consolidated Gas Co.	\$ 100,000
2. Main St. R. R.	\$ 250,000
3. American Steel Co.	\$ 75,000
4. Acme Insurance Co.	\$ 50,000
5. The Harbor Coal Co.	\$ 25,000
6. The Crystal Ice Co.	\$ 37,500

7. Mr. Daly owns 2 houses valued at \$25,000 each. How much does he pay in taxes, if the rate is \$2.12 per \$100 ?

8. Mr. Mayer owns 6 lots valued at \$15,500 each and 2 houses valued at \$3000 each. How much does he pay in taxes, if the tax rate is 2.21 % ?

9. Mr. Holly owns \$3500 worth of personal property and \$15,500 worth of real estate. How much does he pay in taxes, if the tax rate is 2.25 % ?

10. Last year the tax rate was 1.35%. This year it is 1.85%. How much more must be paid this year than last year on \$120,000 worth of property?

11. I own a house worth \$20,000. I spend \$500 a year for repairs and pay taxes of $2\frac{1}{2}\%$. What is my net income from the property during the year, if my tenant pays me \$1500 a year in rent?

DUTIES OR CUSTOMS

FOR READING AND DISCUSSION

188. The National Government or United States Government spends large sums of money each year for the army and navy, for pensions for disabled soldiers and their families, for the large number of government employees, for the United States Courts, etc. The amount spent is more than \$1,000,000 a day. The United States Government does not levy direct taxes on real and personal property; it obtains money for its expenditures by indirect taxation and by an income tax.

One way by which the United States Government obtains money is by taxing goods imported into this country. These taxes are called **Duties** or **Customs**. They are collected by "customs officials" at the great ports of entry, as New York, San Francisco, Boston, etc.

Duties are of two kinds: **specific duties** and **ad valorem duties**.

189. A **specific duty** is a tax levied on **each bushel** or **each pound** or **each gallon**, without regard to the value of the goods.

190. An **ad valorem duty** is, as the name indicates, a tax levied on the **value** of the goods imported.

191. Some goods are subject to both a specific duty and an ad valorem duty. For example, if oilcloth is taxed 12 ct. per square yard, and 15% ad valorem, the tax on 200 sq. yd. of oilcloth worth \$100 would be \$24, specific duty, plus \$15 ad valorem duty, or \$39, total duty.

Some goods are not taxed; they are admitted free of duty.

192. The law stating the different rates and kinds of duties is called the **tariff law**. Tariff laws are passed by Congress and may be changed at any session.

193. **Tare** is an allowance made for the weight of barrels, boxes, bottles, or other covering of goods. **Leakage** is an allowance made for loss of liquids by leakage or evaporation when imported in barrels, casks, etc.

Gross weight is the total weight of the goods and the receptacles in which they are shipped. The **net weight** is the weight after the allowances for tare and leakage have been made. For example, the **gross weight** of 10 kegs of nails is the

total weight of the nails and the kegs; the **tare** is the weight of the kegs; the **net weight** is the gross weight minus the tare.

To find the Specific Duty

194. Suppose the tariff law imposes the following specific duties:

Fruit . . . 25 ct. per bu.	Potatoes . . . 25 ct. per bu.
Chocolate. $2\frac{1}{2}$ ct. per lb.	Raisins . . . $2\frac{1}{2}$ ct. per lb.
Lemons . . $1\frac{1}{2}$ ct. per lb.	Rice . . . $1\frac{1}{2}$ ct. per lb.
Mustard . . 10 ct. per lb.	Salt (in pkgs.) 11 ct. per 100 lb.
Olives . . . 15 ct. per gal.	Vinegar . . . $7\frac{1}{2}$ ct. per gal.

Using the above rates of duty, find the specific duty on:

1. 50 bu. of apples.
2. 100 bu. of pears.
3. 200 bu. of peaches.
4. 300 gal. of olives.
5. 500 boxes of raisins; the net weight of each box being 22 lb.
6. 100 boxes of lemons; the net weight of each box being 40 lb.
7. 100 bags of salt; 100 lb. in each bag.
8. 600 bu. of potatoes.
9. 120 boxes of rice; net weight of each box, 7 lb.
10. 200 bottles of vinegar, each bottle containing 1 gal., allowing 10 % for leakage.
11. The duty on barley was lowered from 30 ct. per bushel to 15 ct. per bushel. How much is saved on an importation of 250 bu.?

12. The duty on lard was lowered from 2 ct. per pound to $1\frac{1}{2}$ ct. per pound. How much is saved on a shipment of 500 lb.?

To find the Ad Valorem Duty

195. To find the ad valorem duty.

Proceed as in the First Type Problem in Percentage (p. 70).

Suppose the tariff law imposes the following ad valorem duties:

Agricultural Implements	15 %	Cutlery	40 %
Boots and Shoes	15 %	Feathers	20 %
Sewing Machines	30 %	Furs	$27\frac{1}{2}$ %
Shoe Laces	35 %	Leather	15 %
Typewriting Machines	40 %	Sponges	30 %

Using the above rates of duty, find the duty on:

1. A shipment of agricultural implements worth \$4000.
2. 60 doz. pairs of boots worth \$2.50 per pair.
3. 10 gross knives worth \$3.50 per dozen, and 10 gross forks worth \$3 per dozen.
4. 150 fine feathers valued at \$6 each.
5. 72 fur coats valued at \$75 each.
6. A shipment of leather worth \$6500.
7. 120 sewing machines worth \$35 each.
8. 70 typewriting machines worth \$48 each.
9. 60 doz. sponges; 50 % of them worth \$3 a dozen; the remainder worth \$5 a dozen.

10. 10 gross shoe laces worth \$ 3.60 a gross.
11. 1 doz. sets of fur worth \$ 300 each.
12. The duty on shoes was lowered from 25 % to 15 %. How much less duty is paid on 80 boxes, each box containing 12 doz. pairs, worth \$ 2.50 per pair?

To find the Duty when both a Specific Duty and an Ad Valorem Duty are Charged

196. Suppose the tariff law imposes the following duties :

- Cocoa, 4 ct. per pound and 10 % ad valorem.
- Ground Spices, 10 ct. per pound and 30 % ad valorem.
- Oilcloth, 12 ct. per square yard and 15 % ad valorem.
- Shirts, 35 ct. per dozen and 10 % ad valorem.
- Stockings, Cotton, 70 ct. per dozen pairs and 15 % ad valorem.

Using the above rates, find the total duty in each of the following :

1. 250 yd. of oilcloth worth 60 ct. per sq. yd.
2. 72 doz. shirts worth \$ 18 per dozen.
3. 120 cans of cocoa, each can containing 1 lb., at 60 ct. per pound.
4. 180 cans of pepper (spices), each can containing $\frac{1}{2}$ lb., at 36 ct. per pound.
5. 288 doz. pairs of cotton stockings, valued at 15 ct. per pair.
6. Obtain the latest tariff schedule. Make and solve four problems based on the schedule.

USE OF THE EQUATION IN THE SOLUTION OF PROBLEMS

Interpretation of Algebraic Expressions

EXERCISES

NOTE TO THE TEACHER. The greatest difficulties in the solution of problems by the Equation Method are found not in the solution of the equation, but in the expression of the conditions and relations in algebraic terms. Continual practice should be given in the interpretation of algebraic expressions (as in art. 197) and in the expression of arithmetical relations in algebraic terms (as in art. 198).

197. 1. If a hat costs \$3, what does $4 \times \$3$ represent?

Say, $4 \times \$3$ represents the cost of 4 hats at \$3 each.

3. If a pound of coffee costs 42 ct., what is represented by 2×42 ct.? By $\frac{1}{2}$ of 42 ct.? By $\frac{1}{4}$ of 42 ct.?

5. A coat costs \$15; a hat costs \$10. What does $\$15 + \10 represent?

2. If a hat costs x dollars, what does $4x$ represent?

Say $4x^*$ represents the cost of 4 hats at x dollars each.

4. If x represents the cost of a pound of coffee, what is represented by $4x$? By $\frac{1}{2}x$? By $\frac{1}{4}x$?

6. A coat costs x dollars; a hat costs \$8. What does $x + 8$ represent?

* The multiplication sign is omitted.

7. A man who had 40 sheep sold 25 of them. What does $40 - 25$ represent?

9. I bought a chair for \$3.75, and gave the storekeeper a \$10 bill in payment. What does $\$10 - \3.75 represent?

11. 12 books cost \$28. What does $\frac{28}{12}$ represent?

13. A hat that cost \$3 is sold at a gain of 25%. What does $\frac{5}{4}$ of \$3 represent?

8. A man who had x sheep sold 20 of them. What does $x - 20$ represent?

10. I bought a chair for x dollars and gave the storekeeper a \$10 bill in payment. What does $\$10 - x$ represent?

12. 12 books cost x dollars. What does $\frac{x}{12}$ represent?

14. A hat that cost x dollars is sold at a gain of 25%. What does $\frac{5}{4}x$ represent?

Representation of Quantities by Symbols

198. 1. One basket costs 60 ct. Represent the cost of 9 baskets.

Say, the cost of 9 baskets is represented by 9×60 ct.

3. A boy who receives \$5 a week (for 6 da.) works only 4 da. Represent the amount of money he receives.

2. A basket cost x ct. Represent the cost of 5 baskets.

Say, the cost of 5 baskets is represented by $5x$.

4. A boy receives x dollars a week (for 6 da.). He works only 3 da. Represent the amount he receives.

5. A man who had \$75 spent \$40. Represent the amount he had left.

6. A man who had \$75 spent x dollars. Represent the amount he had left.

7. Mr. White had \$200 in the bank. He increased this by \$150. Represent the amount now in the bank.

8. Mr. White had \$200 in the bank. He increased this amount by x dollars. How much has he in the bank now?

9. I bought an umbrella for \$5.50 and gave the storekeeper a \$10 bill in payment. Represent the amount I received in change.

10. I bought an umbrella for x dollars and gave the storekeeper a \$20 bill in payment. Represent the amount I received in change.

11. \$50 is spent for shirts at \$2 each. Represent the number of shirts bought.

12. x dollars is spent for shirts at \$2 a dozen. How many shirts are received for the money?

13. 18 lb. of tea cost \$9. Represent the cost of 1 lb.

14. 20 lb. of tea cost x dollars. Represent the cost of 1 lb.

15. One pound of sugar costs 8 ct. Represent the cost of $\frac{1}{2}$ lb. Of $\frac{3}{4}$ lb.

16. One pound of sugar costs x ct. Represent the cost of $\frac{1}{2}$ lb. Of $\frac{3}{4}$ lb.

17. A barrel of apples that cost \$ 3.50 is sold at a gain of 18 %.

(a) Represent the gain.

(b) Represent the selling price.

18. A barrel of apples that cost x dollars is sold at a gain of 18 %.

(a) Represent the gain.

(b) Represent the selling price.

19. If x represents the distance a boat sails the first day, and 100 mi. the distance it sails the second day, represent the distance sailed in both days.

20. A house that cost x dollars is sold at a loss of 9 %.

(a) Represent the loss.

(b) Represent the selling price.

21. \$ 200 is loaned for x yr. at 4 %. Represent the interest.

22. \$ 500 is loaned for x yr. at 5 %. Represent the amount.

NEGATIVE NUMBERS

FOR READING AND DISCUSSION

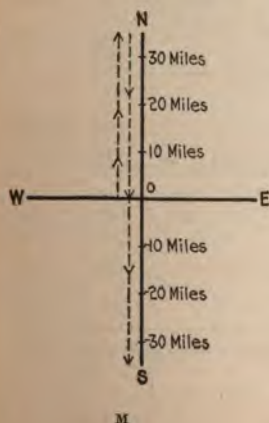
199. In all the problems you have solved up to the present time, the numbers used were **positive** numbers. The lowest number was zero; all the numbers used were greater than zero. But in some problems in arithmetic and algebra, **negative** numbers are used.

The following illustrations show the meaning of negative numbers.

First Illustration: Suppose that the thermometer shows that the temperature is 30° above 0 at 6 o'clock at night. We would represent the temperature by $+30^{\circ}$, meaning 30° above 0. If the weather grew colder, the mercury might fall to 20° , to 10° , and then to 0° . Now suppose the weather grew still colder, so that the mercury fell 10° below zero; we would then say that the temperature was -10° . If the weather continued to grow colder, the temperature would be represented by -20° ; -30° ; etc.



Second Illustration: Suppose a boat sails in a northerly direction from 0. If it sails 10 mi. an hour, its distance from 0 at the end of the first hour would be 10 mi.; at the end of the second hour 20 mi. The distance from the starting point, 0, will increase as long as the boat continues to sail in a northerly direction.



If the vessel turns and sails in a southerly direction, its distance from 0, at the end of the third hour, would be

$$10 + 10 - 10 = 10 \text{ mi. from } 0;$$

at the end of the fourth hour,

$$10 + 10 - 10 - 10 = 0 \text{ mi.},$$

that is, it will be at 0 again.

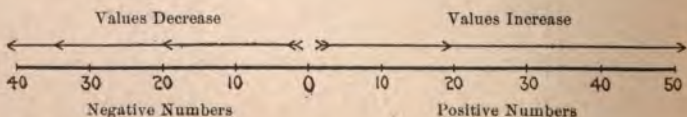
If the vessel continues to sail toward the south, the distance from 0 at the end of the fifth hour would be -10 mi., that is, 10 mi. from 0, measured in a direction opposite to that in which the positive distances were measured. The next hour the vessel would be -20 mi. from 0, etc.

When no sign is written before a quantity, a plus sign is always understood.

There are as many numbers below zero as there are above zero. Numbers above zero are positive numbers; numbers below zero are negative numbers. The values of positive numbers **increase** from 0; the values of negative numbers **decrease** from 0. Thus $+100$ is greater than $+70$, but -100 is less than -70 .

Representation of Positive and Negative Numbers

Positive and negative numbers may be represented by this diagram:



EXERCISES

200. 1. A thermometer registered 20° at 2 P.M.; by 3 P.M. it had risen 10° ; by 5 P.M. it had fallen 20° . What was the temperature at 5 P.M.?

$$20^{\circ} + 10^{\circ} - 20^{\circ} = ?$$

2. A thermometer registered 15° at 5 P.M.; by 6 P.M. it had fallen 10° ; by 7 P.M. it had fallen 15° more. Find the temperature at 7 P.M.

3. A ship sailed from 0 towards the north for 2 hr. at the rate of 10 mi. an hour; then it sailed toward the south for 4 hr. at the rate of 8 mi. an hour. Represent the distance of the ship from 0 at the end of this time.

4. A storekeeper took in \$40 in the morning and \$50 in the afternoon. He paid bills amounting to \$10 and \$15. Represent these transactions by positive and negative numbers.

5. A car started on a trip with 10 passengers. At different stations 15, 25, and 50 people got on the car; also 5, 10, and 20 people left the car. Represent this by positive and negative numbers.

6. A submarine boat was on the surface of the water. It went 5 ft. below the surface; then sank 20 ft. more; then rose 12 ft.; then sank 6 ft.

(a) How far was the boat from the surface of the water?

(b) Represent the movements of the boat by a diagram.

7. An aeroplane starts from the ground and rises 100 ft.; then it sinks 20 ft. and rises 40 ft.; then it sinks 80 ft., sails parallel to the ground for 100 ft., and sinks 40 ft. more.

(a) How far is it from the ground now?

(b) Represent the flight by a diagram.

8. A man had \$1000 in the bank. He deposited \$50, \$70, \$40, and \$200, and withdrew \$20, \$60, \$70, \$100. How much is still in the bank?

9. An agent travels 410 mi. north on Monday, 175 mi. north on Tuesday, 350 mi. south on Wednesday, and 275 mi. south on Thursday. How far was he from the starting point? Illustrate graphically.

201. SOLUTION OF EQUATIONS

$3x - 125 = 275$ is an equation.

$3x$ is the **unknown quantity** or **unknown number**.

$3x - 125$ is the **first member** of the equation.

275 is the **second member** of the equation.

Finding a **value** for x is called **solving** the equation.

202. $3x$ means 3 times x or $3 \times x$; the number 3 is called a **coefficient** of x . When no coefficient is written, the coefficient 1 is understood. x means $1 \times x$ or $1x$.

Important Principles of the Equation

203. 1. Solve: $x - 30 = 75$.

An equation resembles a scale; both sides must be equal.



If 30 lb. are added to the first scale pan, how much must be added to the second scale pan to preserve the balance?

$$x - 30 = 75$$

(Add 30 to each member.)

$$\begin{array}{r} 30 \quad 30 \\ x - 30 = 75 \\ \hline x = 105 \end{array}$$

204. Therefore,

First Principle of the Equation

The same number may be ADDED to each member of an equation without affecting the equality.

2. Solve: $x + 20 = 50$.

Subtract 20 from the first scale pan.

How much is in the first pan now?

How much must be subtracted from the second pan to preserve the equality?

$$x + 20 = 50$$

(Subtract 20 from each member.)

$$\begin{array}{r} 20 \quad 20 \\ x + 20 = 50 \\ \hline x = 30 \end{array}$$

205. Therefore,

Second Principle of the Equation

The same number may be SUBTRACTED from each member of an equation without affecting the equality.

206. In a similar manner the following may be shown :

Third Principle of the Equation

Both members of an equation may be MULTIPLIED by the same number without affecting the equality.

Fourth Principle of the Equation

Both members of an equation may be DIVIDED by the same number without affecting the equality.

EXERCISES

207. Solve each equation. (Prove those marked*.)

*1. $x + 5 = 10$

10. $x - 2 = 0$

2. $x + 9 = 15$

*11. $3x = 15$

*3. $x + 3 = 9$

12. $5x = 30$

4. $x - 12 = 4$

13. $10x = 75$

5. $x - 3 = 8$

*14. $\frac{x}{2} = 6$

6. $x - 5 = 1$

*7. $x + 2 = 19$

8. $x - 9 = 4$

15. $\frac{x}{3} = 15$

*9. $x + 7 = 3$

- | | |
|----------------------------------|---|
| 16. $\frac{x}{4} = 2\frac{1}{2}$ | 23. $\frac{x}{2} + 4 = 5$ |
| 17. $6x = 19$ | 24. $3x + 1 = 22$ |
| *18. $\frac{x}{5} = 10$ | *25. $5x = 17$ |
| 19. $7x = 42$ | 26. $\frac{x}{3} + 4 = 9$ |
| 20. $\frac{x}{6} = 4$ | *27. $2x + 2\frac{1}{2} = 5\frac{1}{2}$ |
| *21. $3x + 5 = 14$ | 28. $\frac{x}{4} + 3 = 7.25$ |
| 22. $2x - 1 = 11$ | 29. $4x - 12 = 27$ |
| | *30. $6x - 2 = 0$ |

PROBLEMS

208. Solve these problems by the Equation Method:

1. 5 blank books cost 35 ct. Find the cost of 1 book.

Let	$x = \text{cost of 1 book.}$
Then	$5x = \text{cost of 5 books,}$
and	$5x = 35 \text{ ct.}$
	$\therefore x = 7 \text{ ct. } \textit{Ans.}$

2. If a boy walks 9 mi. in 3 hr., how far does he walk in 1 hr.?

3. A steamship burns 1800 T. of coal in 5 da. How much coal is used each day?

4. A man is paid \$254 for working 30 da. How much does he receive a day?

5. A machine prints 300 papers in 6 min. How many papers are printed each minute?

6. An expressman must deliver 1200 packages. His wagon can carry only 50 packages. How many trips must be made to deliver all the packages?

7. If 1920 soldiers march 16 abreast, how many rows of soldiers will there be?

8. Find the cost of 1 doz. blank books, if 18 doz. cost \$10.80.

9. A train travels 175 mi. in 7 hr. How far does it travel in an hour?

10. How many baskets are required to hold 456 bottles, if each basket holds 24 bottles?

CLEARING EQUATIONS OF FRACTIONS

209. 1. Find the value of x in the equation $\frac{x}{4} = 16$.

Multiply each member by 4. $x = 64$. *Ans.*
(See principle number 3, p. 166.)

Proof. $\frac{x}{4}$ should equal 16. $\frac{1}{4}$ of $\frac{16}{1} = 16$.

2. Solve: $\frac{x}{3} = 12\frac{1}{2}$.

Multiply each member by 6 (the l. c. m. of the denominators).

$$6 \times \frac{x}{3} = 2x; 6 \times 12\frac{1}{2} = 75.$$

$$2x = 75.$$

$$x = 37\frac{1}{2}. \quad \text{Ans.}$$

Proof. $\frac{x}{3}$ should equal $12\frac{1}{2}$.

210. RULE. To clear equations of fractions : **MULTIPLY EVERY TERM** in the equation by the **L. C. M.** of the **denominators** of the fractions.

EXERCISES

211. Clear these equations of fractions ; then find values for x . (Prove those marked *.)

$$*1. \quad \frac{x}{2} = 15$$

$$*10. \quad \frac{2x}{5} = 6\frac{1}{2}$$

$$2. \quad \frac{x}{3} = 8$$

$$11. \quad \frac{4x}{9} = 15$$

$$3. \quad \frac{x}{5} = 4$$

$$12. \quad 2x + \frac{x}{2} = 5$$

$$4. \quad \frac{2x}{3} = 6$$

$$*13. \quad 2x - \frac{x}{3} = 10$$

$$*5. \quad \frac{5x}{6} = 20$$

$$14. \quad 3x - \frac{x}{2} = 14$$

$$6. \quad \frac{3x}{2} = 9$$

$$15. \quad 6x - \frac{2x}{3} = 17$$

$$7. \quad \frac{x}{2} = 4\frac{1}{2}$$

$$16. \quad \frac{9x}{10} = 72$$

$$8. \quad \frac{x}{4} = 3\frac{1}{2}$$

$$*17. \quad 1.25x = 15$$

$$18. \quad 2.50x = 25$$

$$9. \quad \frac{x}{7} = 1\frac{1}{4}$$

$$19. \quad 1.10x = 33$$

$$*20. \quad 3\frac{1}{2}x = 21$$

$$21. \quad \frac{x}{2} + 1.25x = 3\frac{1}{2}$$

PROBLEMS

212. Solve the following problems by the Equation Method :

1. $\frac{1}{2}$ lb. of tea costs 25 ct. Find the cost of 1 lb.
2. $\frac{1}{4}$ lb. of pepper costs 9 ct. Find the cost of 1 lb.
3. $\frac{2}{3}$ of the height of a building is 90 ft. How high is the building?
4. $\frac{3}{4}$ of the number of boys in a school is 336. How many boys are there in the school?
5. $\frac{5}{6}$ of the weight of a machine is 65 lb. Find the weight of the machine.
6. $3\frac{1}{2}$ yd. of ribbon cost 21 ct. Find the cost of 1 yd.
7. If a man earns \$30 in $\frac{2}{3}$ of a week, how much will he earn in a week?
8. $\frac{1}{2}$ the length of a room is $15\frac{1}{2}$ ft. Find the length of the room.
9. $\frac{3}{4}$ of the amount of the money collected for a present is \$82 $\frac{1}{2}$. How much money was collected?
10. The money gained by selling a desk is \$8 $\frac{1}{2}$. This is $33\frac{1}{3}\%$ of the cost. Find the cost.
11. The advance in the price of meat was $7\frac{1}{2}$ ct. a pound. This was 25% of its former price. Find the former price.
12. The width of a plot of ground is 225% of the length. The width is 81 ft. Find the length.

13. The number of bushels of wheat produced on a field this year is $12\frac{1}{2}\%$ greater than last year. If the number produced this year is 1260 bu., how many bushels were produced last year?

14. By using a new kind of furnace, a company required $16\frac{2}{3}\%$ less coal this year than last year. If the company used 240 T. this year, how many tons were used last year?

15. The number of girls in a school is $\frac{1}{2}$ as great as the number of boys. The total number of pupils is 600. How many girls are there? How many boys?

TRANSPOSING TERMS IN AN EQUATION

213. The **first member** of an equation should contain only **unknown quantities**; the **second member** should contain only **known quantities**.

For example, in $3x = 21$, $3x$ is an unknown quantity and is in the first member; 21 is a known quantity and is in the second member.

Frequently equations contain quantities that are not in the proper member, as

$$3x + 5 = 26, \text{ or } 3x - 7 = x + 21.$$

Here 5 should be in the second member; 7 should be in the second member; x should be in the first member.

Subtract 5 from each member

(Principle 2, p. 166).

$$\begin{array}{r} 3x + 5 = 26 \\ \quad \quad \quad 5 \quad 5 \\ \hline 3x \quad \quad = 26 - 5 \end{array}$$

We see that 5 may be changed or **transposed** from the first to the second member by **changing its sign**.

Add 7 to each member (Principle 1, p. 165).

(Subtract x from each member.)

$$\begin{array}{r} 3x - 7 = x + 21 \\ \quad \quad \quad 7 \quad \quad \quad 7 \\ \hline 3x \quad \quad = x + 21 + 7 \\ \quad \quad \quad x \quad \quad \quad x \\ \hline 3x - x = 21 + 7 \\ \text{or} \\ 2x = 28 \end{array}$$

7 is transposed from the first to the second member by changing its sign.

x is transposed from the second to the first member by changing its sign.

Therefore,

214. RULE FOR TRANSPOSING TERMS. Any term may be transposed from one member of an equation to the other, provided its sign is changed.

215. Solve the following equations. (Prove those marked*.)

*1. $x + 4 = 7$

8. $4x - 2 = x + 7$

2. $x - 6 = 15$

9. $5x + 6 = 2x + 19$

3. $2x + 3 = 9$

*10. $3x - 5 = 8 - 2x$

4. $3x - 5 = 10$

11. $\frac{x}{2} + 3 = 15$

*6. $3x = 10 + x$

12. $\frac{3x}{4} - 2 = 10$

7. $2x + 5 = x + 9$

$$*13. \quad \frac{2x}{3} + \frac{1}{2} = \frac{x}{2} + 9$$

$$14. \quad 2x + \frac{3}{4} = x + 6$$

$$15. \quad 5x + 2 = \frac{x}{2} + 4$$

$$16. \quad 10x = 28 - 3x$$

$$*17. \quad 4x + \frac{1}{2} = 2x + 1$$

$$18. \quad 3x + 350 = 2x + 500$$

$$19. \quad 3x + 9 = \frac{x}{3} + 12$$

$$*20. \quad 5x - 6 = \frac{2x}{3} + 20$$

PROBLEMS

216. Solve the following problems by the Equation Method:

1. William and John have \$200; John has \$50. How much money has William?

2. 5 new pupils were admitted to a class, making 42 pupils on register. How many pupils were in the class at first?

3. 6 pupils left a class, making the register 40. How many pupils were in the class at first?

4. The weight of a statue and a wooden base is 25 lb. The base weighs 6 lb. Find the weight of the statue.

5. A man drew \$250 from a bank and had \$625 left. How much did he have in the bank originally?

6. The total distance sailed by a ship in two days was 235 mi. If it sailed 110 mi. the first day, how many miles did it sail the second day?

7. William and John have \$200; John has \$50 more than William. How much money has each?

8. A log of wood was 18 ft. long. A piece was cut off, leaving the log $12\frac{1}{2}$ ft. long. Find the length of the piece that was cut off.

9. A piece of wood 7 ft. long was cut from a log, leaving the log $10\frac{1}{2}$ ft. long. How long was the log before the piece was cut off?

10. A log was cut into two pieces, one $5\frac{1}{2}$ ft. long, the other $9\frac{3}{4}$ ft. long. How long was the log before it was cut?

11. The length of a string was increased to 16 ft. by tying on a piece $4\frac{1}{2}$ ft. long. How long was the string originally?

12. The population of a city increased 75,000 during the last 10 yr., thereby making the population 650,000. Find the population of 10 yr. ago.

13. A train left New York with a certain number of passengers. At the first station 60 more people got on the train. There were then 110 people on the train. How many were on the train when it left New York?

14. 48 pupils graduated from a school and 100 new pupils were admitted. After these changes the register was 900. How many pupils were on register before the changes had been made?

15. A wagon that weighed 125 lb. carried 40 packages. The total weight of the wagon and all the packages is 325 lb. Find the weight of each package.

SIMPLE INTEREST

FOR READING AND DISCUSSION

217. In business it often becomes necessary to borrow money. Banks, trust companies, and individuals make business of lending money to persons offering good security. For the use of this money interest is paid. Moreover, banks and trust companies themselves pay interest to depositors on savings and balances left on deposit for the required time.

218. Money for the use of which interest is paid is called the **principal**.

The **rate of interest** is the number of hundredths paid for the use of the principal for 1 yr.

The number of years, months, and days for which interest is paid is called the **time**.

Money paid for the use of the principal is called **interest**.

The **amount** is the sum of the principal and interest.

NOTE. The most common rate of interest is 6 %. Money is frequently loaned at 3 %, 4 %, 5 %, or $5\frac{1}{2}$ %. Savings banks usually pay 3 %, $3\frac{1}{2}$ %, or 4 % interest. Large sums of money are sometimes loaned at low rates of interest, *e.g.* 2 % or $2\frac{1}{2}$ %.

In many states it is illegal to charge more than 6 % interest. The charging of more than the legal rate of interest is called **usury**.

TO FIND THE SIMPLE INTEREST AND THE AMOUNT

The Aliquot Parts Method for Years and Months

219. 1. Find the simple interest and amount of \$ 800 for 3 yr. at 6 %.

1st Process

\$ 800	
.06	
<hr/>	
\$ 48.00	Int. for 1 yr. at 6 %.
3	
<hr/>	
\$ 144.00	Int. for 3 yr. at 6 %.
\$ 800 + \$ 144 = \$ 944,	Amt.
	<i>Ans.</i>

2d Process

Int. for 1 yr. = .06 of principal.	
Int. for 3 yr. = .18 of principal.	
\$ 800	
.18	
<hr/>	
\$ 144	Int. for 3 yr. at 6 %.
\$ 800 + \$ 144 = \$ 944,	Amt.
	<i>Ans.</i>

RULE. To find the Interest :

Interest = Principal \times Rate \times Number of Years.

2. Find the interest and amount of \$ 3000 for 1 yr. 3 mo. at 4 %.

1st Process

1 yr. 3 mo. = $1\frac{1}{4}$ yr.	
\$ 3000	
.04	
<hr/>	
\$ 120	Int. for 1 yr.
30	Int. for 3 mo. or $\frac{1}{4}$ yr.
<hr/>	
\$ 150	Int. for 1 yr. 3 mo. <i>Ans.</i>
\$ 3000 + \$ 150 = \$ 3150,	Amt.

2d Process

1 yr. 3 mo. = $1\frac{1}{4}$ yr.	
Int. at 4 % for $1\frac{1}{4}$ yr. = .05 of principal.	
\$ 3000	
.05	
<hr/>	
\$ 150	Int. <i>Ans.</i>
\$ 3000 + \$ 150 = \$ 3150,	Amt.

Aliquot Parts Method applied to Days

220. 1. Find the interest on \$ 1200 for 2 yr. 6 mo. 15 da. at 4 %.

TO FIND SIMPLE INTEREST AND AMOUNT 177

Process

Int. on \$1200 at 4% for 1 yr.	= \$ 48
Int. on \$1200 at 4% for 1 yr. more	= 48
Int. on \$1200 at 4% for 6 mo. = $\frac{1}{2}$ of \$48	= 24
Int. on \$1200 at 4% for 15 da. = $\frac{1}{2}$ of $\frac{1}{2}$ of \$48 =	2
Int. for 2 yr. 6 mo. 15 da.	\$122. <i>Ans.</i>

NOTE. For computing interest, a month contains 30 da.

ORAL EXERCISES

221. Find the interest on \$100 at 6%

- | | | |
|---------------------------|---------------------------|--------------------------|
| 1. For 1 yr. | 4. For $2\frac{1}{2}$ yr. | 7. For $\frac{1}{2}$ yr. |
| 2. For 2 yr. | 5. For 5 yr. | 8. For $\frac{3}{4}$ yr. |
| 3. For $1\frac{1}{2}$ yr. | 6. For $3\frac{1}{2}$ yr. | 9. For $\frac{2}{3}$ yr. |

10. Find the interest on \$200 for the time and rate indicated in exercises 1-9.

11. Find the interest on \$100 for the time indicated in exercises 1-9 at 4%.

EXERCISES

222. Find the interest on :

Principal	Time	Rate	Principal	Time	Rate
1. \$600	1 yr.	5%.	8. \$600	1 yr.	2%.
2. \$800	1 yr.	6%.	9. \$600	1 yr.	$5\frac{1}{2}$ %.
3. \$1200	1 yr.	3%.	10. \$350	2 yr.	$3\frac{1}{2}$ %.
4. \$200	1 yr.	2%.	11. \$900	1 yr.	4%.
5. \$700	1 yr.	4%.	12. \$900	3 yr.	4%.
6. \$800	2 yr.	3%.	13. \$900	2 yr.	6%.
7. \$800	2 yr.	5%.	14. \$900	3 yr.	6%.

6. An expressman must deliver 1200 packages. His wagon can carry only 50 packages. How many trips must be made to deliver all the packages?

7. If 1920 soldiers march 16 abreast, how many rows of soldiers will there be?

8. Find the cost of 1 doz. blank books, if 18 doz. cost \$10.50.

9. A train travels 175 mi. in 7 hr. How far does it travel in an hour?

10. How many baskets are required to hold 456 bottles if each basket holds 24 bottles?

CLEARING EQUATIONS OF FRACTIONS

319. Find the value of x in the equation

4

Multiply each member by 4 $x = 64$ Ans.

The principle number 3, p. 166.

Check: $\frac{1}{4}$ of 64 = 16

4 $\frac{1}{4} = 1 \frac{3}{4}$

Multiply each member by 4 the L. c. m. of the denomi-

$$3 \cdot \frac{1}{4} = \frac{3}{4} \quad 3 \cdot \frac{1}{2} = \frac{3}{2} \quad 3 \cdot \frac{1}{3} = 1$$

$$\frac{3}{4} = \frac{3}{4}$$

$$\frac{3}{2} = \frac{3}{2}$$

$$\frac{3}{4} = \frac{3}{4}$$

210. RULE. To clear equations of fractions : **MULTIPLY EVERY TERM** in the equation by the **L. C. M.** of the denominators of the fractions.

EXERCISES

211. Clear these equations of fractions; then find values for x . (Prove those marked *.)

$$*1. \quad \frac{x}{2} = 15$$

$$*10. \quad \frac{2x}{5} = 6\frac{1}{2}$$

$$2. \quad \frac{x}{3} = 8$$

$$11. \quad \frac{4x}{9} = 15$$

$$3. \quad \frac{x}{5} = 4$$

$$12. \quad 2x + \frac{x}{2} = 5$$

$$4. \quad \frac{2x}{3} = 6$$

$$*13. \quad 2x - \frac{x}{3} = 10$$

$$*5. \quad \frac{5x}{6} = 20$$

$$14. \quad 3x - \frac{x}{2} = 14$$

$$6. \quad \frac{3x}{2} = 9$$

$$15. \quad 6x - \frac{2x}{3} = 17$$

$$7. \quad \frac{x}{2} = 4\frac{1}{2}$$

$$16. \quad \frac{9x}{10} = 72$$

$$8. \quad \frac{x}{4} = 3\frac{1}{2}$$

$$*17. \quad 1.25x = 15$$

$$18. \quad 2.50x = 25$$

$$19. \quad 1.10x = 33$$

$$9. \quad \frac{x}{7} = 1\frac{1}{4}$$

$$*20. \quad 3\frac{1}{2}x = 21$$

$$21. \quad \frac{x}{2} + 1.25x = 3\frac{1}{2}$$

	Principal	Time	Rate		Principal	Time	Rate
15.	\$ 900	3 yr.	5 %.	18.	\$ 2000	4 yr.	3 %.
16.	\$ 2500	3 yr.	3 %.	19.	\$ 2000	4 yr.	4 %.
17.	\$ 1500	2 yr.	5 %.	20.	\$ 2000	4 yr.	5 %.

EXERCISES

223. Find the interest and amount on each of the following :

	Principal	Time		Rate		Principal	Time		Rate
		Yr.	Mo.				Yr.	Mo.	
1.	\$ 500	2	6	5 %.	11.	\$ 935	3	6	5 %.
2.	\$ 750	2	3	6 %.	12.	\$ 760	4	2	4 %.
3.	\$ 1050	2	4	4 %.	13.	\$ 275	3	8	6 %.
4.	\$ 295	2	8	3 %.	14.	\$ 415	2	1	3 %.
5.	\$ 350	2	9	2 %.	15.	\$ 490	3	9	7 %.
6.	\$ 820	1	4	5 %.	16.	\$ 310	1	2	2 %.
7.	\$ 560	1	8	6 %.	17.	\$ 1345	3	1	3 %.
8.	\$ 290	1	9	3 %.	18.	\$ 1960	1	4	6 %.
9.	\$ 2750	4	2	5 %.	19.	\$ 2475	3	5	4 %.
10.	\$ 6750	1	7	6 %.	20.	\$ 3150	2	11	6 %.

EXERCISES

224. Find the interest at 6 % on :

- \$ 600 for 1 yr. 2 mo. 10 da.
- \$ 900 for 2 yr. 3 mo. 5 da.

Find the interest at 4 % on :

- \$ 2000 for 1 yr. 2 mo. 10 da.
- \$ 3000 for 1 yr. 8 mo. 27 da.

EXERCISES

225. Find the interest and amount in each of the following: (See note.)

1. On \$2400 from May 1, 1909, to May 1, 1912, at 4 %.
2. On \$1800 from May 1, 1909, to Nov. 1, 1912, at 6 %.
3. On \$1250 from Jan. 15, 1904, to Aug. 15, 1907.
4. On \$2100 from April 10, 1909, to Oct. 10, 1912.
5. On \$790 from Nov. 20, 1910, to Jan. 10, 1913.
6. On \$475 from Oct. 1, 1908, to April 1, 1911.
7. On \$1340 from Aug. 25, 1905, to Dec. 25, 1909.
8. On \$2650 from Feb. 14, 1910, to Jan. 14, 1914.
9. On \$3275 from Dec. 20, 1910, to June 20, 1913.
10. On \$5500 from Sept. 25, 1910, to July 25, 1914.
11. On \$4.50 from March 1, 1913, to Dec. 24, 1914.
12. Make and solve an original exercise in finding interest and amount.

NOTE. Methods of finding the difference between dates are shown on page 46 and in the appendix.

Cancellation Method

226. 1. Find the interest on \$ 900 for 3 mo. 15 da. at 4 %.

Process

Change time to days, counting 30 da. to the month, or 360 da. to the year.

$$\begin{array}{rcl}
 3 \text{ mo.} & = & 90 \text{ da.} \\
 15 \text{ da.} & = & 15 \text{ da.} \\
 \text{Total time} & = & 105 \text{ da.}
 \end{array}
 \quad
 \begin{array}{r}
 9 \\
 \$900 \times \frac{4}{100} \times \frac{105}{360} = \$10.50. \quad \text{Ans.} \\
 21 \\
 40 \\
 10 \\
 2
 \end{array}$$

The rate is written as a common fraction ($\frac{4}{100}$). Do not reduce the rate to lowest terms.

2. Find the interest on \$ 225 for 47 da. at 5 %.

Process

$$\begin{array}{r}
 2.25 \\
 \$225 \times \frac{5}{100} \times \frac{47}{360} = \frac{\$105.75}{72} = \$1.468+ \text{ or } \$1.47. \quad \text{Ans.} \\
 72
 \end{array}$$

NOTE. 100 in the denominator is canceled by pointing off two decimal places to the left in the numerator.

EXERCISES

227. Find the interest on \$ 500 at 6 % for

- | | | |
|-------------|------------|-------------|
| 1. 60 days | 4. 45 days | 7. 36 days |
| 2. 90 days | 5. 75 days | 8. 50 days |
| 3. 120 days | 6. 20 days | 9. 130 days |

Find the interest on \$ 1235 at 4 % for

- | | | |
|-------------|--------------|--------------|
| 10. 15 days | 13. 120 days | 16. 51 days |
| 11. 25 days | 14. 42 days | 17. 150 days |
| 12. 72 days | 15. 10 days | 18. 240 days |

Find the interest in each of the following :

19. \$ 1500 for 36 da., at 4 %.
20. \$ 2400 for 84 da., at 6 %.
21. \$ 3000 for 120 da., at 3 %.
22. \$ 2700 for 350 da., at 5 %.
23. \$ 1600 for 72 da., at $3\frac{1}{2}$ %.
24. \$ 425.60 for 160 da., at $2\frac{1}{2}$ %.
25. \$ 360 for 180 da., at 5 %.
26. \$ 240 for 6 mo. 15 da., at 6 %.
27. \$ 720 for 8 mo. 12 da., at 4 %.
28. \$ 950 for 4 mo. 15 da., at 5 %.
29. \$ 1425 for 6 mo. 10 da., at 5 %.
30. \$ 320.25 for 2 mo. 20 da., at 4 %.
31. \$ 510.75 for 4 mo. 12 da., at 4 %.
32. \$ 825.50 for 9 mo. 20 da., at 3 %.
33. \$ 1500 for 2 mo. 16 da., at 3 %.
34. \$ 2750 for 7 mo. 12 da., at 6 %.

	Principal	Rate	Time
* 35.	\$ 125	4 %	From Jan. 15 to Dec. 10.
36.	\$ 640	6 %	From Dec. 20 to July 15.
37.	\$ 335	4 %	From Aug. 16 to March 20.
38.	\$ 728	5 %	From Feb. 20 to Nov. 25.

* For methods of finding difference between dates, see page 46 and the appendix.

	Principal	Rate	Time
39.	\$ 960	6 %	From July 15 to Feb. 20.
40.	\$ 340	5 %	From Oct. 10 to May 12.
41.	\$ 685	$3\frac{1}{2}$ %	From March 25 to Dec. 15.
42.	\$ 1215	$4\frac{1}{2}$ %	From Sept. 1 to March 15.
43.	\$ 395	6 %	From April 12 to Oct. 22.
44.	\$ 275	$5\frac{1}{2}$ %	From May 18 to Sept. 30.

EXERCISES

228. Find the interest and the amount on :

	Principal	Rate	Time
1.	\$ 800	4 %	From Jan. 5, 1909, to Nov. 26, 1912.
2.	\$ 1100	6 %	From Oct. 20, 1911, to Jan. 15, 1915.
3.	\$ 1950	5 %	From March 10, 1908, to Dec. 23, 1911.
4.	\$ 2225	3 %	From Dec. 15, 1910, to March 12, 1913.
5.	\$ 740	$3\frac{1}{2}$ %	From Feb. 20, 1909, to Oct. 20, 1912.
6.	\$ 1290	$4\frac{1}{2}$ %	From July 10, 1912, to Dec. 15, 1915.
7.	\$ 1500	$2\frac{1}{2}$ %	From Nov. 10, 1910, to May 20, 1913.
8.	\$ 975.50	$5\frac{1}{2}$ %	From April 25, 1910, to Sept. 15, 1912.

TO FIND SIMPLE INTEREST AND AMOUNT 183

Principal	Rate	Time
9. \$650.25	6%	From May 5, 1907, to April 10, 1910.

Six Per Cent Method

229. 1. Find the interest on \$500 for 2 yr. 6 mo. 18 da. at 6%.

This method is based on the following:

At 6%, the interest on \$1 for 1 yr. is \$.06.

At 6%, the interest on \$1 for 1 mo. is $\frac{1}{12}$ of \$.06 or \$.005.

At 6%, the interest on \$1 for 1 da. is $\frac{1}{360}$ of \$.005 or \$.000 $\frac{1}{8}$.

Process

Interest on \$1 for 2 yr. = $2 \times \$.06 = \$.12$

Interest on \$1 for 6 mo. = $6 \times \$.005 = \$.03$

Interest on \$1 for 18 da. = $18 \times \$.000\frac{1}{8} = \$.003$

Interest on \$1 for 2 yr. 6 mo. 18 da. = $\$.153$

Interest on \$500 for 2 yr. 6 mo. 18 da. = $500 \times \$.153$
= \$76.50. *Ans.*

2. Find the interest on \$400 for 4 yr. 3 mo. 24 da. at 4%.

Process

Interest on \$1 for 4 yr. = $4 \times \$.06 = \$.24$

Interest on \$1 for 3 mo. = $3 \times \$.005 = \$.015$

Interest on \$1 for 24 da. = $24 \times \$.000\frac{1}{8} = \$.004$

Interest on \$1 for 4 yr. 3 mo. 24 da. at 6% = $\$.259$

Interest on \$400 for 4 yr. 3 mo. 24 da. at 6% = $400 \times \$.259 = \103.60 .

Therefore, interest at 4% = $\frac{2}{3}$ of \$103.60 = \$69.06 $\frac{2}{3}$ or \$69.07. *Ans.*

RULE. For finding interest by the Six Per Cent Method:

First: Find the interest on \$1 for the given time at 6%.

Second: Multiply the interest on \$1 for the given time at 6%, by the principal.

Third: For rates other than 6%, proceed as in Art. 230.

230. For rates other than 6% :

For 2%, take $\frac{1}{3}$ of the interest at 6%.

For 3%, take $\frac{1}{2}$ of the interest at 6%.

For 4%, subtract $\frac{1}{3}$ of the interest at 6%, or take $\frac{2}{3}$ of the interest at 6%.

For 5%, subtract $\frac{1}{6}$ of the interest at 6%, or take $\frac{5}{6}$ of the interest at 6%.

EXERCISES

231. Find the interest in the following by the Six Per Cent Method:

	Principal	Time	Rate
1.	\$ 1400	2 yr. 6 mo.	6 %
2.	\$ 2700	4 yr. 3 mo.	6 %
3.	\$ 2300	2 yr. 8 mo.	3 %
4.	\$ 1950	3 yr. 4 mo.	2 %
5.	\$ 1200	1 yr. 6 mo. 12 da.	4 %
6.	\$ 2600	2 yr. 3 mo. 18 da.	4 %
7.	\$ 3200	2 yr. 2 mo. 24 da.	5 %
8.	\$ 4500	1 yr. 3 mo. 6 da.	5 %
9.	\$ 1370	2 yr. 2 mo. 21 da.	$3\frac{1}{2}$ %

PROBLEMS

232. 1. How much greater is the interest on \$2000 for 2 yr. at 6 % than at 4 % ?

2. How much greater is the interest on \$1500 for 2 yr. 6 mo. at 6 % than at 3 % ?

3. How much less is the interest on \$1250 for 3 yr. 2 mo. at 3 % than at 5 % ?

4. Mr. Jones buys a house worth \$25,000. He pays \$15,000 cash and gives a mortgage for the balance at 4 %. How much interest is paid each 6 mo. ?

5. I loan \$60,000 for 2 yr. On $\frac{2}{3}$ of it I receive 4 % interest ; on the balance I receive 5 % interest. Find the total interest.

6. How much greater is the interest on \$3600 for 3 yr. 6 mo. at 6 % than for 2 yr. 3 mo. at 6 % ?

7. A savings bank lowered the rate of interest it paid from 4 % to $3\frac{1}{2}$ % per annum. How much less interest is paid for 6 mo. to a man who has \$1950 on deposit ?

8. A company had been paying 6 % interest on \$250,000 each year. It borrowed \$250,000 from a man who was willing to accept only $4\frac{1}{2}$ % interest. How much will the company save in 5 yr. at the lower rate of interest ?

9. I loaned \$1200 to Mr. Furness on July 16, 1909. The money was repaid on Oct. 24, 1911, with interest at 4 %. How much was repaid ?

10. 360 tables worth \$ 15 each were bought on May 20, 1909. The bill was paid on Nov. 25, 1909, with interest at 6 %. Find the amount paid.

11. A bill for \$ 500 should have been paid on April 20, 1911. It was paid on Aug. 30, 1911, with interest at 5 %. How much was paid?

12. A debt of \$ 2500 was due on March 15, 1909. It was paid on Sept. 30, 1910, with interest at 4 %. How much was paid on Sept. 30, 1910?

13. A man bought \$ 2000 worth of goods on May 1, 1910. If he had paid cash, he would have received a discount of 2 %. Instead of paying cash, he paid the debt on Oct. 30, 1910, with interest at 4 %. How much did he lose by not paying cash?

14. A man bought 10 lots at an average price of \$ 1500 per lot. He paid $\frac{1}{3}$ of the cost in cash, and allowed the balance to remain on mortgage at $5\frac{1}{2}$ % per annum. How much interest will he pay on the mortgage in 3 yr.?

EXACT INTEREST OR ACCURATE INTEREST

233. The United States Government computes interest on the basis of 365 da. to the year. This interest is called Exact Interest or Accurate Interest.

Exact interest is usually found by the "Cancellation Method." See page 180.

1. Find the exact interest on \$ 450 for 135 da. at 4 %.

$$\text{Process } \$ \overset{4.50}{\cancel{450}} \times \frac{4}{100} \times \frac{\overset{27}{\cancel{135}}}{\underset{73}{365}} = \$ 6.657 \text{ or } \$ 6.66.$$

Note that the denominator of the fraction indicating the time is 365 instead of 360.

EXERCISES

Find the exact or accurate interest on:

1. \$ 320 for 140 da. at 6 %.
2. At 4 %.
3. \$ 540 for 300 da. at 4 %.
4. At 5 %.
5. \$ 160.40 from May 26, 1913, to Oct. 9, 1915, at 5 %.
6. \$ 1750 from April 5, 1912, to Oct. 19, 1916, at 2 %.
7. \$ 2570 from Jan. 10, 1909, to July 31, 1914, at $3\frac{1}{2}$ %.
8. How much less is the exact interest on \$ 2250 for 100 da. at 6 % than the simple interest for the same time at the same rate?
9. If you borrowed money, would you prefer to pay simple interest on the basis of 360 da. to a year, or exact interest? Why?
10. Find the exact interest on the examples in Art. 227.

INDIRECT CASES OF INTEREST

To find the Rate Per Cent when the Principal,
Interest, and Time are Given

234. 1. At what rate must \$300 be loaned, to yield \$42 interest in 2 yr. 4 mo.?

This problem corresponds to type problem No. 5, on page 11.

Exercises 1, 2, and 3, Art. 235, may be solved by the use of the equation. Let $\frac{x}{100}$ or $\frac{r}{100}$ represent the rate per cent.

Process

Interest on \$300 for 2 yr. 4 mo. at 1% = \$7.*

Therefore, the rate per cent required to yield \$42 interest is as much as $\$42 \div \7 or 6. 6%. *Ans.*

RULE. To find the Rate when the Principal, Interest, and Time are given: Divide the GIVEN INTEREST by the INTEREST on the given principal for the given time at 1%.

EQUATION. Rate = Interest \div Interest on Given Principal at 1%.

EXERCISES

235. 1. At what rate will \$300 yield \$67.50 interest in 4 yr. 6 mo.?

2. At what rate will \$450 yield \$76.50 interest in 2 yr. 10 mo.?

3. At what rate will \$480 yield \$50.40 interest in 2 yr. 4 mo.?

4. At what rate will \$540 yield \$85.59 interest in 2 yr. 7 mo. 21 da.?

* First find 1% of \$300; then multiply by 2 $\frac{1}{2}$.

5. At what rate will \$155 double itself in 16 yr. and 8 mo.?

6. At what rate will any sum double itself in 16 yr. and 8 mo.?

7. At what rate will \$150 double itself in 14 yr.?

8. At what rate will any sum double itself in 14 yr.?

9. Interest \$221.44, principal \$69.20, time 8 mo. 16 da. Find the rate.

10. At what rate will \$2250 yield \$648 interest in 6 yr. 4 mo. 24 da.?

11. At what rate will \$880 earn \$82.28 interest in 1 yr. 8 mo. 12 da.?

12. At what rate will \$570 earn \$54.34 interest in 2 yr. 11 mo. 6 da.?

13. At what rate will \$880 earn \$73.04 interest from May 14, 1910, to Oct. 2, 1911?

14. A man borrowed \$810 on Sept. 24, 1909. On April 2, 1913, he paid \$142.65 interest. What was the rate?

15. A merchant borrowed \$1260 on Aug. 20, 1908. On Dec. 16, 1910, he paid \$1391.67 in settlement of the debt. What rate of interest did he pay?

16. Solve exercises 4, 5, and 6 by use of the equation.

To find the Time when the Principal, Interest, and Rate Per Cent are Given

236. 1. In what time will \$ 450 yield \$ 63.90 interest at 6 % ?

Process

This problem corresponds to type problem No. 5, on page 11.

Exercises 1-10 may be solved by the use of the equation. What letter may be used for the unknown quantity in these equations ?

Interest on \$ 450 for 1 yr. at 6 % = \$ 27.

Therefore, the number of years required to yield \$ 63.90 interest is as many as $\$ 63.90 \div \$ 27 = 2\frac{11}{30}$.

$2\frac{11}{30}$ yr. = 2 yr. 4 mo. 12 da.

Ans.

RULE. — To find the Time :

Divide the **GIVEN INTEREST** by the **INTEREST** on the given principal for 1 YEAR at the given rate per cent.

EQUATION. Time = Interest \div Interest on Given Principal at Given Rate for 1 YEAR.

EXERCISES

237. 1. In what time will \$ 5000 yield \$ 350 interest @ 6 % ?

2. In what time will \$ 480 yield \$ 56 interest @ 5 % ?

3. In what time will \$ 350 yield \$ 30.80 interest @ 4 % ?

4. In what time will \$ 750 yield \$ 20.25 interest @ $4\frac{1}{2}$ % ?

5. In what time will \$ 375 yield \$ 74.25 interest @ $5\frac{1}{2}$ % ?

6. In what time will \$420 yield \$36.75 interest @ $3\frac{1}{3}\%$?

7. In what time will \$6500 yield \$614.25 interest @ $3\frac{1}{2}\%$?

8. A man borrowed \$4175 on May 1, 1906, at $4\frac{1}{2}\%$ interest. On the day of settlement, he paid \$4609.20. What was the date of payment?

9. On May 26, 1907, a man obtained a mortgage for \$33,000 at 5% interest. On settlement, \$36,960 was paid. When was the mortgage paid?

10. A debt of \$1150 was incurred on Aug. 4, 1910. On settlement, \$1301.80 was paid, including the interest at $5\frac{1}{2}\%$. Find the date of settlement.

To find the Principal when the Interest, the Rate Per Cent, and the Time are Given

238. 1. I loaned a sum of money for 6 mo. at 6%. I received \$12 interest. Find the sum loaned (principal).

Process

This problem corresponds to type problem No. 5, page 11.

In exercises 1-11, Art. 238, try to use the equation.

Interest on \$1 for 6 mo. at 6% = \$.03.

Therefore, the number of dollars that will yield \$12 interest is as many as $\$12 \div \$.03$ or \$400. *Ans.*

2. What principal will yield \$82.20 interest in 2 yr. 3 mo. 12 da. at 5%?

Process

Interest on \$1 for 2 yr. 3 mo. 12 da. at 6% = \$.137.

Interest on \$1 for 2 yr. 3 mo. 12 da. at 5% = \$.114 $\frac{1}{8}$.

Therefore, number of dollars that will yield \$82.20 interest at 5% is

$$\$82.20 \div \$.114\frac{1}{8}.$$

Multiplying dividend and divisor by 6

$$\$493.20 \div .685 = \$720 \text{ Principal. } \textit{Ans.}$$

239. RULE. To find the Principal: Divide the GIVEN INTEREST by the INTEREST ON \$1 for the given time at the given rate.

EQUATION. Principal = Interest \div (INTEREST ON \$1 for given time at given rate).

EXERCISES

240. Find the principal in each of the following:

	Time	Rate	Interest
1.	3 yr.,	6 %,	\$180.
2.	2 yr.,	4 %,	\$96.
3.	2 yr. 6 mo.,	4 %,	\$150.
4.	2 yr. 4 mo.,	6 %,	\$105.
5.	1 yr. 10 mo.,	5 %,	\$82.50.
6.	1 yr. 6 mo.,	3 %,	\$36.
7.	1 yr. 3 mo.,	3 $\frac{1}{2}$ %,	\$70.
8.	3 yr. 3 mo.,	4 %,	\$312.
9.	2 yr. 8 mo.,	5 %,	\$280.
10.	1 yr. 3 mo.,	2 %,	\$47.50.
11.	1 yr. 6 mo. 12 da.,	6 %,	\$184.

- | | Time | Rate | Interest |
|-----|---------------------|--------------------|------------|
| 12. | 1 yr. 8 mo. 24 da., | 3 %, | \$650. |
| 13. | 2 yr. 6 mo. 12 da., | 4 %, | \$1672. |
| 14. | 2 yr. 4 mo. 18 da., | $4\frac{1}{2}\%$, | \$2145. |
| 15. | 3 yr. 9 mo. 21 da., | 5 %, | \$3541.75. |
16. From March 2, 1912, to June 6, 1912, 6 %, interest, \$ 80.
17. From May 25, 1911, to Dec. 11, 1911, 4 %, interest, \$ 60.
18. From Jan. 4, 1912, to Nov. 8, 1912, 3 %, interest, \$ 123.60.
19. From Feb. 20, 1910, to July 8, 1912, 2 %, interest, \$ 171.60.
20. From May 16, 1910, to March 22, 1913, 4 %, interest, \$ 342.

241. To find the Principal when the Amount, Rate Per Cent, and Time are Given

1. I loaned a sum of money for 1 yr. 6 mo. at 6 %. When the debt was paid, I received \$ 218. Find the sum loaned (principal).

This problem corresponds to type problem Number 5 on page 11.

Process

\$ 1 at 6 % for 1 yr. 6 mo. amounts to \$ 1.09. Therefore, the number of dollars that will amount to \$ 218 is as many as $\$ 218 \div \$ 1.09$ or 200. \$ 200 *Ans.*

242. RULE. To find the Principal when the Amount, Rate Per Cent, and Time are Given:

Divide the GIVEN AMOUNT by the AMOUNT OF \$1 for the given time, at the given rate.

EQUATION. $\text{Principal} = \text{Amount} \div \text{AMOUNT OF \$1 for given time at given rate.}$

EXERCISES

243. Find the principal in each of the following:

Time	Rate	Amount
1. 1 yr. 6 mo.,	6 %,	\$ 490.50.
2. 1 yr. 3 mo.,	4 %,	\$ 819.
3. 1 yr. 2 mo.,	3 %,	\$ 952.20.
4. 2 yr. 3 mo. 12 da.,	6 %,	\$ 1705.50.
5. 2 yr. 4 mo. 18 da.,	4 %,	\$ 1971.60.
6. 3 yr. 6 mo. 6 da.,	6 %,	\$ 2906.40.
7. From Jan. 5, 1910, to July 20, 1912, 6 %, amount, \$ 1452.15.		
8. From May 1, 1908, to Aug. 5, 1911, 4 %, amount, \$ 2034.80.		
9. From July 9, 1911, to Aug. 18, 1913, 5 %, amount, \$ 795.90.		
10. From March 4, 1912, to Aug. 20, 1914, $3\frac{1}{2}$ %, amount, \$ 8211.21.		

REVIEW EXERCISES IN INTEREST

244. Find values for ?.

	PRINCIPAL	RATE	TIME			INTEREST	AMOUNT
			yr.	mo.	da.		
1.	\$ 1240	$4\frac{1}{2}\%$	2	5	18	?	?
2.	\$ 675	?	3	6	20	\$ 144.00	—
3.	\$ 990	4 %		?		\$ 90.09	—
4.	?	$2\frac{1}{2}\%$	1	7	6	\$ 40.40	—
5.	?	4 %	2	2	18	—	\$ 1355.39
6.	\$ 450	4 %	2	7	6	?	?
7.	\$ 468	?	1	7	22	\$ 38.48	—
8.	\$ 760	$4\frac{1}{2}\%$?		\$ 61.18	—
9.	?	6 %	1	8	15	\$ 31.16	—
10.	?	$2\frac{1}{2}\%$	2	8	24	—	\$ 2211.45
11.	\$ 1240	$2\frac{1}{2}\%$	2	10	15	—	?
12.	\$ 672	?	3	3	5	—	\$ 737.80
13.	—	5 %		?		\$ 85.40	\$ 925.40
14.	?	4 %	2	2	18	\$ 110.39	—
15.	?	$3\frac{1}{2}\%$	1	7	6	—	\$ 554.40
16.	\$ 780	4 %	1	10	15	—	?
17.	\$ 450	?	2	4	24	—	\$ 487.80
18.	—	4 %		?		\$ 47.77	\$ 812.77
19.	?	$4\frac{1}{2}\%$	2	0	20	\$ 67.34	—
20.	?	3 %	2	0	24	—	\$ 3748.86
21.	\$ 1320	$3\frac{1}{2}\%$	3	3	6	?	?
22.	—	?	1	3	26	\$ 33.32	\$ 453.32
23.	\$ 432	$3\frac{1}{2}\%$?		—	\$ 459.30
24.	?	4 %	2	5	1	\$ 113.23	—
25.	?	5 %	3	10	24	—	\$ 924.93
26.	\$ 3720	3 %	3	3	21	?	?
27.	—	?	1	7	13	\$ 34.98	\$ 574.98
28.	\$ 560	3 %		?		—	\$ 620.34

245. COMPUTING SIMPLE INTEREST BY TABLES

THIS TABLE SHOWS THE NUMBER OF DAYS FROM ANY DAY OF ANY MONTH TO THE SAME DAY OF ANY MONTH NOT MORE THAN ONE YEAR LATER

FROM	TO JAN.	FEB.	MAR.	APR.	MAY	JUNE	JULY	AUG.	SEPT.	OCT.	NOV.	DEC.
Jan.	365	31	59	90	120	151	181	212	243	273	304	334
Feb.	334	365	28	59	89	120	150	181	212	242	273	303
Mar.	306	337	365	31	61	92	122	153	184	214	245	275
April	275	306	334	365	30	61	91	122	153	183	214	244
May	245	276	304	335	365	30	61	91	123	153	184	214
June	214	245	273	304	334	365	30	61	92	122	153	183
July	184	215	243	274	304	335	365	31	62	92	123	153
Aug.	153	184	212	243	273	304	334	365	31	61	92	122
Sept.	122	153	181	212	242	273	303	334	365	30	61	91
Oct.	92	123	151	182	212	243	273	304	335	365	31	61
Nov.	61	92	120	151	181	212	242	273	304	334	365	30
Dec.	31	62	90	121	151	182	212	243	274	304	335	365

EXERCISES

246. 1. Find the number of days from May 10 to Dec. 15 of the same year. Use the table.

Process

By the table, from May 10 to Dec. 10 = 214 da.

From Dec. 10 to 15 = 5 da.

Total 219 da.

2. Find the number of days from July 1 to Dec. 24 of the same year.

3. Find the number of days from Feb. 20 to Sept. 19 of the same year.

247. SIMPLE INTEREST TABLE. INTEREST AT 6%

Da.	\$100	\$200	\$300	\$400	\$500	\$600	\$700	\$800	\$900	\$1000	Da.
1	0.017	0.033	0.050	0.067	0.083	0.100	0.117	0.133	0.150	0.167	1
2	0.033	0.067	0.100	0.133	0.167	0.200	0.233	0.267	0.300	0.333	2
3	0.050	0.100	0.150	0.200	0.250	0.300	0.350	0.400	0.450	0.500	3
4	0.067	0.133	0.200	0.267	0.333	0.400	0.467	0.533	0.600	0.667	4
5	0.083	0.167	0.250	0.333	0.417	0.500	0.583	0.667	0.750	0.833	5
6	0.100	0.200	0.300	0.400	0.500	0.600	0.700	0.800	0.900	1.000	6
7	0.117	0.233	0.350	0.467	0.583	0.700	0.817	0.933	1.050	1.167	7
8	0.133	0.267	0.400	0.533	0.667	0.800	0.933	1.067	1.200	1.333	8
9	0.150	0.300	0.450	0.600	0.750	0.900	1.050	1.200	1.350	1.500	9
10	0.167	0.333	0.500	0.667	0.833	1.000	1.167	1.333	1.500	1.667	10
11	0.183	0.367	0.550	0.733	0.917	1.100	1.283	1.467	1.650	1.833	11
12	0.200	0.400	0.600	0.800	1.000	1.200	1.400	1.600	1.800	2.000	12
13	0.217	0.433	0.650	0.867	1.083	1.300	1.517	1.733	1.950	2.167	13
14	0.233	0.467	0.700	0.933	1.167	1.400	1.633	1.867	2.100	2.333	14
15	0.250	0.500	0.750	1.000	1.250	1.500	1.750	2.000	2.250	2.500	15
16	0.267	0.533	0.800	1.067	1.333	1.600	1.867	2.133	2.400	2.667	16
17	0.283	0.567	0.850	1.133	1.417	1.700	1.983	2.267	2.550	2.833	17
18	0.300	0.600	0.900	1.200	1.500	1.800	2.100	2.400	2.700	3.000	18
19	0.317	0.633	0.950	1.267	1.583	1.900	2.217	2.533	2.850	3.167	19
20	0.333	0.667	1.000	1.333	1.667	2.000	2.333	2.667	3.000	3.333	20
21	0.350	0.700	1.050	1.400	1.750	2.100	2.450	2.800	3.150	3.500	21
22	0.367	0.733	1.100	1.467	1.833	2.200	2.567	2.933	3.300	3.667	22
23	0.383	0.767	1.150	1.533	1.917	2.300	2.683	3.067	3.450	3.833	23
24	0.400	0.800	1.200	1.600	2.000	2.400	2.800	3.200	3.600	4.000	24
25	0.417	0.833	1.250	1.667	2.083	2.500	2.917	3.333	3.750	4.167	25
26	0.433	0.867	1.300	1.733	2.167	2.600	3.033	3.467	3.900	4.333	26
27	0.450	0.900	1.350	1.800	2.250	2.700	3.150	3.600	4.050	4.500	27
28	0.467	0.933	1.400	1.867	2.333	2.800	3.267	3.733	4.200	4.667	28
29	0.483	0.967	1.450	1.933	2.417	2.900	3.384	3.867	4.350	4.833	29
Mo.	\$100	\$200	\$300	\$400	\$500	\$600	\$700	\$800	\$900	\$1000	Mo.
1	0.500	1.000	1.500	2.000	2.500	3.000	3.500	4.000	4.500	5.000	1
2	1.000	2.000	3.000	4.000	5.000	6.000	7.000	8.000	9.000	10.000	2
3	1.500	3.000	4.500	6.000	7.500	9.000	10.500	12.000	13.500	15.000	3
4	2.000	4.000	6.000	8.000	10.000	12.000	14.000	16.000	18.000	20.000	4
5	2.500	5.000	7.500	10.000	12.500	15.000	17.500	20.000	22.500	25.000	5
6	3.000	6.000	9.000	12.000	15.000	18.000	21.000	24.000	27.000	30.000	6
7	3.500	7.000	10.500	14.000	17.500	21.000	24.500	28.000	31.500	35.000	7
8	4.000	8.000	12.000	16.000	20.000	24.000	28.000	32.000	36.000	40.000	8
9	4.500	9.000	13.500	18.000	22.500	27.000	31.500	36.000	40.500	45.000	9
10	5.000	10.000	15.000	20.000	25.000	30.000	35.000	40.000	45.000	50.000	10
11	5.500	11.000	16.500	22.000	27.500	33.000	38.500	44.000	49.500	55.000	11
Yr.	\$100	\$200	\$300	\$400	\$500	\$600	\$700	\$800	\$900	\$1000	Yr.
1	6.00	12.00	18.00	24.00	30.00	36.00	42.00	48.00	54.00	60.00	1
2	12.00	24.00	36.00	48.00	60.00	72.00	84.00	96.00	108.00	120.00	2
3	18.00	36.00	54.00	72.00	90.00	108.00	126.00	144.00	162.00	180.00	3
4	24.00	48.00	72.00	96.00	120.00	144.00	168.00	192.00	216.00	240.00	4
5	30.00	60.00	90.00	120.00	150.00	180.00	210.00	240.00	270.00	300.00	5

EXERCISES IN USING TABLES

248. 1. Find the interest on \$ 650 from Jan. 4, 1913, to May 26, 1913.

Process

From the tables, from Jan. 4 to May 4 = 120 da.

From May 4 to May 26 = 22 da.

Total = 144 da. = 4 mo.
22 da.

From the tables, interest on

$$\$600 \text{ for } 4 \text{ mo. at } 6\% = \$12.00$$
$$\$50 \text{ for 4 mo. at } 6\% = 1.00$$
$$\$600 \text{ for } 22 \text{ da. at } 6\% = 2.20$$

\$50 for 22 da. at 6% = .183

Total interest at 6% = \$15.383. *Ans.*

By using the tables, find the interest on following:

2. \$ 450 for 2 yr. 7 mo. at 6 %.
3. \$ 630 from March 25 to Oct. 30 at 6 %.
4. \$ 740 for 1 yr. 5 mo. at 6 %.
5. \$ 840 from May 12, 1911, to Feb. 19, 1911,
at 6 %.
6. \$ 420 for 2 yr. 3 mo. 5 da. at 6 %.
7. \$ 525 from June 8 to Dec. 5 at 4 %.
8. \$ 520 for 2 yr. 7 mo. 12 da. at 3 %.
9. \$ 1200 from Jan. 9 to Nov. 26 at 3 %.
10. \$ 360 for 127 da. at 6 %.
11. \$ 2750 from May 25 to Dec. 19 at 5 %.

12. \$ 1500 for 220 da. at 4 %.
13. \$ 3500 from March 4 to Nov. 16 at 6 %.
14. \$ 3200 for 92 da. at 4 %.
15. \$ 1450 from Feb. 15, 1910, to Jan. 5, 1912, at 6 %.
16. \$ 1750 for 128 da. at 5 %.
17. \$ 2840 from March 18 to July 22 at 3 %.
18. \$ 1375 for 116 da. at 3 %.
19. By using the tables find the interest on the examples in Art. 222.
20. In Art. 225.
21. In Art. 227.

PROMISSORY NOTES

249. A promissory note is an unconditional promise to pay a specified sum of money on a specified day.

Form I, Non-negotiable note.

\$ 276 ⁰⁰ / ₁₀₀	Buffalo, N. Y., March 17, 1913
Three months	after date I promise to pay to
the order of James Lane Allen	
Two Hundred Seventy-six	500 Dollars
at his office, 225 Fifth Avenue, New York	
Value received	
No. 872 Due June 17, 1913	Horace Mann

Form II, Interest-bearing note; negotiable without indorsement.

\$ 50 ⁰⁰ / ₁₀₀	Akron, Ohio, June 15, 1913	
Ninety days	after date	I promise to pay to
the order of Bearer		
Fifty		⁰⁰ / ₁₀₀ Dollars
at The Fifth National Bank, Akron, Ohio		
Value received with interest at 5%		
No. 224	Due Sept. 13, 1913	John Henderson

Form III, Demand note, negotiable by indorsement.

\$ 100 ⁰⁰ / ₁₀₀	New York, N. Y., May 20, 1913	
On demand	after date	we promise to pay to
the order of Charles E. Field		
One Hundred		⁰⁰ / ₁₀₀ Dollars
at our office		
Value received		
No. 901	Due	Allen and Hayes J. L. A.

250. The **face** of a note is the specified amount of money.

251. There are two principal "parties" or "persons" to a note, the **maker** and the **payee**.

The **maker** is the party who signs the note.

The **payee** is the party to whom the money is to be paid.

252. The **date of maturity** is the day on which the note becomes due and payable.

In times past, it was a custom sanctioned by law to allow the maker of a note three days beyond the specified time for payment before resorting to process to compel payment. These three days are known as "**days of grace.**" With the improved means of communication afforded by the railroad and the telegraph, the custom is dying out. It has already been abolished in most of the states.

QUERY. Does the law allow days of grace in your state?

A **demand note**, Form III, is due and payable on any day that the holder chooses to present it for payment. Some of our paper money consists of "demand notes" made "legal tender" by statute.

Interest-bearing notes contain the words "with interest." If the rate is not specified, the legal rate is charged.

A non-interest-bearing note, if "dishonored" upon maturity, draws interest at the legal rate from maturity until settlement.

The words "value received" usually appear upon a note.

If the place of payment is not specified, the note must be presented for payment at the place of business or at the residence of the maker.

253. A **negotiable note** is one that may be transferred by the holder to another party.

The holder is the party that holds the note.

254. **Indorsement** is the signing of an order on the back of a note before transferring the note to another party.

The party that indorses a note is called an indorser.

A note drawn payable "to bearer" is negotiable without indorsement.

A note drawn payable "to the order of" the payee is negotiable by "indorsement."

A note that does not contain the words "to bearer" or "to the order of" is not negotiable; *i.e.* it is payable only to the party named in the note as payee.

255. There are several kinds of indorsement.

Indorsement in full assigns the note to a specified person and guarantees the payment of the note.

Pay to the order of
Robert Blair.

CHAS. E. FIELD.

Indorsement in blank assigns the note "to bearer" and guarantees its payment. The note is thereafter negotiable without indorsement.

CHAS. E. FIELD.

Restrictive indorsement converts a negotiable note into a non-negotiable note.

Pay to Robert Blair.
CHAS. E. FIELD.

Indorsement "without recourse" discharges the indorser from liability for non-payment of the note. It is sometimes called a "qualified indorsement."

Without recourse.
CHAS. E. FIELD.

If the maker fails to pay the note when it is presented for payment upon maturity, the note is said to be **dishonored**.

In order to take advantage of the guarantee of payment made by previous indorsers of the note, the holder of a dishonored note must have the note "protested" and must serve "notice" on the previous indorsers.

256. Protest is the declaration made before a notary public that the note has been presented to the maker for payment and that payment has been refused.

257. Notice is the notification of protest sent to the indorsers of a note within twenty-four hours after the day of maturity.

258. To find the date of maturity of a note, the number of months or of days, as expressed in the note, is to be counted forward from the date of the note.

To illustrate, a three months' note dated Dec. 31, 1912, is due on March 31, 1913; and a ninety days' note is also due March 31; but a two months' note of Dec. 31, 1912, would be due Feb. 28, 1913, and a sixty days' note on March 1, 1913.

EXERCISES

259. 1. Make the following non-negotiable, non-interest-bearing notes, writing before the date the place in which the note is signed:

Date	Maker	Payee	Face	Term	Place of Pay't
Sept. 5, 19—	Yourself	R. F. Adams	\$ 308.61	2 mo.	Payee's office
Jan. 4, 19—	T. Y. Loftus	Yourself	\$ 5000.00	90 da.	Your office

2. Write a negotiable note for \$ 645.40 payable by J. F. Andrews to you sixty days after date, bearing the legal rate of interest.

3. Write a negotiable non-interest-bearing note for \$ 1000 payable by yourself to Henry Stoddard six months after date, at the maker's office.

4. Write a non-negotiable note: maker, yourself; payee, Jacob Schmidt; make it payable sixty days after date, with interest at 5 per cent.

5. Write a negotiable demand note for \$ 225, and indorse it in full to J. H. Brown.

6. Write a non-negotiable demand note for \$ 850 payable by F. P. Adams to yourself, without interest. Write an indorsement "without recourse."

7. Make a negotiable note for \$ 1250 in payment of merchandise sold to Wright and Evans by yourself, payable in sixty days.

BANK DISCOUNT

FOR READING AND DISCUSSION

260. The business of buying notes and other negotiable paper is largely committed to commercial banks. When a merchant borrows money from a bank, he usually gives a negotiable, non-interest-bearing note for the time agreed upon. The bank discounts the note at the rate of interest agreed upon, usually the legal rate, and pays to him the

face of the note less the interest or discount. In computing the **bank discount** (or interest) the value of the note at maturity is taken as the base. In the case of an interest-bearing note, the **amount** of the note is the base. In other words, the bank receives **interest in advance** upon the money value of the note when due.

261. **Bank discount** is the simple interest paid in advance upon the amount due on a note or draft at maturity.

262. The **proceeds** of a note is the amount of the note or draft at maturity less the bank discount.

If a note does not bear interest, the amount is the face of the note.

263. The **term of the note** is the time between the date of the note and its maturity.

264. The **term of discount** is the number of days that the note has to run from the day of discount to the day of maturity.

In states that allow days of grace, three days additional should be included in the "term of discount" and in the "term of the note."

265. Suppose that Henry Thompson has received the following note from John A. Stewart in payment of a bill of goods. He may keep the note until it matures and then present it to Mr. Stewart

for payment; or he may keep it for a part of the time and then discount it at the bank; or he may discount it the very day that he receives it.

\$ 1000 ⁰⁰ / ₁₀₀	Tacoma, Wash., May 25	1913
Sixty days	after date I promise to pay to	
the order of Henry Thompson		
One Thousand		00 Dollars
at the State Bank of Tacoma		
Value received		
No 121	Due July 24, 1913	John O. Stewart

Let us suppose that the note is discounted on May 25, at 6 %.

The **date of maturity** is sixty days after May 25, which is July 24.

The **term of discount** is 60 da.

The **bank discount** is the simple interest on \$ 1000 for 60 da. at 6 %, or \$ 10.

The **proceeds** is \$ 1000 minus \$ 10, or \$ 990.

NOTE. If the above note were to read "Two months after date" instead of "Sixty days after date," the date of maturity would be July 25, and the term of discount 61 da. The term of discount is always computed as the exact number of days from discount till maturity.

If the above note were discounted on June 1, the term of discount would be 29 da. + 24 da., or 53 da., the bank discount \$ 8.83, and the proceeds \$ 981.17.

If the above note were made in Alabama or in some other state which allows days of grace, the date of maturity would be July 27 and the term of discount 64 da.

EXERCISES

266. Find the bank discount and the proceeds of the following notes :

1. For \$ 3500, payable in 90 da., discounted at 6 %.

2. For \$ 5000, payable in 30 da., discounted at 9 %.

3. For \$ 150, payable in 60 da., discounted at $4\frac{1}{2}$ %.

4. For \$ 500, payable in 42 da., discounted at 7 %.

5. For \$ 640, payable in 33 da., discounted at 5 %.

6. For \$ 400, payable Dec. 31, discounted Nov. 13 at 6 %.

7. For \$ 1500, payable on March 15, 1912, discounted Jan. 28, 1912, at 8 %.

8. For \$ 2150, payable on Jan. 3, discounted on Aug. 21 at $7\frac{1}{2}$ %.

9. For \$ 875, dated April 19, due Aug. 19, discounted on date drawn at 5 %.

10. Find the date of maturity, the term of discount, the bank discount, and the proceeds of a note for \$ 600, drawn June 18, payable in 120 da., discounted on June 18 at $4\frac{1}{2}$ %.

Find the proceeds of the following notes :

11. \$ 300, drawn Jan. 15, 1913, payable in 60 da., discounted on Feb. 1, 1913, at 4 %.

12. \$ 750, drawn Oct. 24, payable in 3 mo., discounted on Oct. 24 at 6 %.

13. For \$ 75, drawn July 5, payable in 2 mo., discounted on July 15 at 6 %.

14. For \$ 2800, drawn Dec. 31, 1912, payable in 2 mo., discounted on Jan. 9, 1913, at $4\frac{1}{2}$ %.

15. For \$ 2800, drawn Dec. 31, 1912, payable in 60 da., discounted on Jan. 9, 1913, at $4\frac{1}{2}$ %.

16. For \$ 2000, with interest at 5 %, for 60 da., discounted on date drawn at 5 %.

HINT. The amount of the note at maturity is the base on which bank discount is computed.

17. For \$ 375, with interest at 7 %, for 90 da., discounted on date drawn at 8 %.

18. For \$ 1250, with interest at $4\frac{1}{2}$ %, dated Aug. 10, payable in 30 da., discounted Aug. 15 at 5 %.

19. For \$ 1800, with interest at 6 %, dated Sept. 12, payable in 3 mo., discounted Sept. 18 at 6 %.

HINT. Compute the "interest" for 3 mo., and the "discount" for the *exact number of days in the term of discount*.

20. For \$10,000, with interest at 7 %, dated Jan. 4, 1912, payable in 6 mo., discounted Feb. 1, 1912, at 5 %.

21. For \$340.75, with interest at 5 %, dated Nov. 1, 1913, payable in 4 mo., discounted Feb. 4 at 7 %.

22. For \$58,647.29, with interest at 3 %, dated Nov. 30, 1913, payable in 3 mo., discounted at 4 % on day made.

23. For \$200,000, with interest at $2\frac{1}{2}$ %, dated July 31, payable in 2 mo., discounted Aug. 30 at $5\frac{1}{2}$ %.

This note was made in Indiana, which allows 3 da. of grace.

To find the Face of a Note

267. For what sum must I draw my note at 60 da. to obtain \$200 in cash, if the rate of discount is 6 % ?

Process	Explanation
$\$1 - .01 = .99 =$ proceeds of \$1.	Since 99 ct. is the proceeds of a note for \$1,
$\$200 \div \$.99 = 202.02$	
$\$1 \times 202.02 = \$202.02.$ Ans.	\$200 is the proceeds of a note for as many dollars as 99 ct. is contained times in \$200, which is 202.02 times.
Solve problems 1-7, Art. 268, by use of the equation.	$202.02 \times \$1 = \$202.02.$

~~SECRET~~

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... .. gain by
paying

MULTIPLES AND POWERS

269. Compare these expressions :

1. $3 \times 5 = 15$.

2. $5 \times 5 = 25$.

3. $2 \times 2 \times 3 \times 5 = 60$.

4. $5 \times 5 \times 5 = 125$.

In the first expression, 15 is a multiple of 5, and of 3. In the second expression, 25 is a multiple of 5. In the third expression, 60 is a multiple of 2, of 3, and of 5. In the fourth expression, 125 is a multiple of 5.

A **multiple** of a number is the **product** obtained by multiplying that number by another number.

15 is a multiple of 5 and 3; 30 is a multiple of 6 and 5.

270. In the second expression, 25 is obtained by multiplying 5 by **itself**, or by taking 5 as a factor twice. In the fourth expression, 125 is obtained by multiplying 5 by itself twice, or by taking 5 as a factor three times.

25 and 125 are called **powers** of 5.

A **power** of a number is the **product** obtained by multiplying that number by itself a certain number of times.

271. Since $2 \times 2 = 4$, **4** is the **second power** of 2.

$2 \times 2 \times 2 = 8$, **8** is the **third power** of 2.

$4 \times 4 = 16$, **16** is the **second power** of 4.

$4 \times 4 \times 4 = 64$, **64** is the **third power** of 4.

The second power of a number is called the square of the number.

The third power of a number is called the cube of the number.

The 10th power is the 10th power, 10th power, and powers are named from the number of times a number is raised as a factor.

272. Defining the Name Multiple and Power

A Multiple is the Power is the name
result obtained by given to those products
multiplying a number only that arise from the
by any other number. multiplication of a num-
ber by itself, a certain
number of times.

273 The power to which a number is to be
raised is indicated by a small figure called the
exponent.

314 3 2 1 2 3 4 The small 2 is the exponent.

2 3 2 3 2 3 2 3 2 3 The small 3 is the ex-
ponent.

274 To raise a fraction to a given power, raise
both the numerator and the denominator to the
given power.

$$\frac{1}{2}^2 = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

$$\frac{3}{4}^2 = \frac{3^2}{4^2} = \frac{9}{16}$$

275 Any power of 1 is 1, because 1 \times 1 any
number of times always equals 1.

Every number is considered to be the first power of itself; the exponent 1 is not written.

276. To **square** a number is to raise it to the second power; 4 is the square of 2.

To **cube** a number is to raise it to the third power; 8 is the cube of 2.

EXERCISES

- 277.** 1. Square 2; 3; 5; 8; 4; 10; 12
 2. Cube 2; 3; 5; 6; 4; 1; 7

EXERCISES

278. Raise each number to the power indicated by the exponent.

- | | | | |
|--------------|-----------|---------------|-----------|
| 1. $2^2 = ?$ | $2^3 = ?$ | 6. $5^2 = ?$ | $5^3 = ?$ |
| 2. $2^4 = ?$ | $2^5 = ?$ | 7. $6^2 = ?$ | $6^3 = ?$ |
| 3. $3^2 = ?$ | $3^3 = ?$ | 8. $7^2 = ?$ | $7^3 = ?$ |
| 4. $3^4 = ?$ | $3^5 = ?$ | 9. $8^2 = ?$ | $8^3 = ?$ |
| 5. $4^2 = ?$ | $4^3 = ?$ | 10. $9^2 = ?$ | $9^3 = ?$ |

EXERCISES

279. Find values for :

	Number	Second Power	Third Power		Number	Second Power	Third Power
1.	9	?	?	6.	30	?	?
2.	12	?	?	7.	50	?	?
3.	15	?	?	8.	10	?	?
4.	20	?	?	9.	100	?	?
5.	25	?	?	10.	1000	?	?

for payment; or he may keep it for a part of the time and then discount it at the bank; or he may discount it the very day that he receives it.

\$ 1000 ⁰⁰ / ₁₀₀	Sacramento, Wash., May 25	1913
Sixty days	after date, I promise to pay to	
the order of Henry Thompson		
One Thousand		00/100 Dollars
at the State Bank of Sacramento		
Value received		
No. 127	Due July 24, 1913	John A. Stewart

Let us suppose that the note is discounted on May 25, at 6 %.

The **date of maturity** is sixty days after May 25, which is July 24.

The **term of discount** is 60 da.

The **bank discount** is the simple interest on \$ 1000 for 60 da. at 6 %, or \$ 10.

The **proceeds** is \$ 1000 minus \$ 10, or \$ 990.

NOTE. If the above note were to read "Two months after date" instead of "Sixty days after date," the date of maturity would be July 25, and the term of discount 61 da. The term of discount is always computed as the exact number of days from discount till maturity.

If the above note were discounted on June 1, the term of discount would be 29 da. + 24 da., or 53 da., the bank discount \$ 8.83, and the proceeds \$ 981.17.

If the above note were made in Alabama or in some other state which allows days of grace, the date of maturity would be July 27 and the term of discount 64 da.

EXERCISES

266. Find the bank discount and the proceeds of the following notes:

1. For \$ 3500, payable in 90 da., discounted at 6 %.

2. For \$ 5000, payable in 30 da., discounted at 9 %.

3. For \$ 150, payable in 60 da., discounted at $4\frac{1}{2}$ %.

4. For \$ 500, payable in 42 da., discounted at 7 %.

5. For \$ 640, payable in 33 da., discounted at 5 %.

6. For \$ 400, payable Dec. 31, discounted Nov. 13 at 6 %.

7. For \$ 1500, payable on March 15, 1912, discounted Jan. 28, 1912, at 8 %.

8. For \$ 2150, payable on Jan. 3, discounted on Aug. 21 at $7\frac{1}{2}$ %.

9. For \$ 875, dated April 19, due Aug. 19, discounted on date drawn at 5 %.

10. Find the date of maturity, the term of discount, the bank discount, and the proceeds of a note for \$ 600, drawn June 18, payable in 120 da., discounted on June 18 at $4\frac{1}{2}$ %.

Find the proceeds of the following notes :

11. \$ 300, drawn Jan. 15, 1913, payable in 60 da., discounted on Feb. 1, 1913, at 4 %.

12. \$ 750, drawn Oct. 24, payable in 3 mo., discounted on Oct. 24 at 6 %.

13. For \$ 75, drawn July 5, payable in 2 mo., discounted on July 15 at 6 %.

14. For \$ 2800, drawn Dec. 31, 1912, payable in 2 mo., discounted on Jan. 9, 1913, at $4\frac{1}{2}$ %.

15. For \$ 2800, drawn Dec. 31, 1912, payable in 60 da., discounted on Jan. 9, 1913, at $4\frac{1}{2}$ %.

16. For \$ 2000, with interest at 5 %, for 60 da., discounted on date drawn at 5 %.

HINT. The amount of the note at maturity is the base on which bank discount is computed.

17. For \$ 375, with interest at 7 %, for 90 da., discounted on date drawn at 8 %.

18. For \$ 1250, with interest at $4\frac{1}{2}$ %, dated Aug. 10, payable in 30 da., discounted Aug. 15 at 5 %.

19. For \$ 1800, with interest at 6 %, dated Sept. 12, payable in 3 mo., discounted Sept. 18 at 6 %.

HINT. Compute the "interest" for 3 mo., and the "discount" for the *exact number of days in the term of discount*.

20. For \$10,000, with interest at 7 %, dated Jan. 4, 1912, payable in 6 mo., discounted Feb. 1, 1912, at 5 %.

21. For \$340.75, with interest at 5 %, dated Nov. 1, 1913, payable in 4 mo., discounted Feb. 4 at 7 %.

22. For \$58,647.29, with interest at 3 %, dated Nov. 30, 1913, payable in 3 mo., discounted at 4 % on day made.

23. For \$200,000, with interest at $2\frac{1}{2}$ %, dated July 31, payable in 2 mo., discounted Aug. 30 at $5\frac{1}{2}$ %.

This note was made in Indiana, which allows 3 da. of grace.

To find the Face of a Note

267. For what sum must I draw my note at 60 da. to obtain \$200 in cash, if the rate of discount is 6 % ?

Process	Explanation
$\$1 - .01 = .99 =$ proceeds of \$1.	Since 99 ct. is the proceeds of a note for \$1,
$\$200 \div \$.99 = 202.02$	\$200 is the proceeds of
$\$1 \times 202.02 = \$202.02.$ <i>Ans.</i>	a note for as many dollars as 99 ct. is contained
Solve problems 1-7, Art. 268,	times in \$200, which is
by use of the equation.	202.02 times.
	$202.02 \times \$1 = \$202.02.$

Find the proceeds of the following notes :

11. \$ 300, drawn Jan. 15, 1913, payable in 60 da., discounted on Feb. 1, 1913, at 4 %.

12. \$ 750, drawn Oct. 24, payable in 3 mo., discounted on Oct. 24 at 6 %.

13. For \$ 75, drawn July 5, payable in 2 mo., discounted on July 15 at 6 %.

14. For \$ 2800, drawn Dec. 31, 1912, payable in 2 mo., discounted on Jan. 9, 1913, at $4\frac{1}{2}$ %.

15. For \$ 2800, drawn Dec. 31, 1912, payable in 60 da., discounted on Jan. 9, 1913, at $4\frac{1}{2}$ %.

16. For \$ 2000, with interest at 5 %, for 60 da., discounted on date drawn at 5 %.

HINT. The amount of the note at maturity is the base on which bank discount is computed.

17. For \$ 375, with interest at 7 %, for 90 da., discounted on date drawn at 8 %.

18. For \$ 1250, with interest at $4\frac{1}{2}$ %, dated Aug. 10, payable in 30 da., discounted Aug. 15 at 5 %.

19. For \$ 1800, with interest at 6 %, dated Sept. 12, payable in 3 mo., discounted Sept. 18 at 6 %.

HINT. Compute the "interest" for 3 mo., and the "discount" for the *exact number of days in the term of discount*.

20. For \$10,000, with interest at 7 %, dated Jan. 4, 1912, payable in 6 mo., discounted Feb. 1, 1912, at 5 %.

21. For \$340.75, with interest at 5 %, dated Nov. 1, 1913, payable in 4 mo., discounted Feb. 4 at 7 %.

22. For \$58,647.29, with interest at 3 %, dated Nov. 30, 1913, payable in 3 mo., discounted at 4 % on day made.

23. For \$200,000, with interest at $2\frac{1}{2}$ %, dated July 31, payable in 2 mo., discounted Aug. 30 at $5\frac{1}{2}$ %.

This note was made in Indiana, which allows 3 da. of grace.

To find the Face of a Note

267. For what sum must I draw my note at 60 da. to obtain \$200 in cash, if the rate of discount is 6 % ?

Process	Explanation
$\$1 - .01 = .99 =$ proceeds of \$1.	Since 99 ct. is the pro-
$\$200 \div \$.99 = 202.02$	ceeds of a note for \$1,
$\$1 \times 202.02 = \$202.02.$ <i>Ans.</i>	\$200 is the proceeds of
Solve problems 1-7, Art. 268,	a note for as many dol-
by use of the equation.	lars as 99 ct. is contained
	times in \$200, which is
	202.02 times.
	$202.02 \times \$1 = \$202.02.$

EXERCISES

268. 1. For how much must a note at 90 da. be drawn so as to produce \$ 1000 when discounted at 5 % ?

2. I wish to borrow \$ 500 from the bank on a 30-da. note. For what sum must I draw the note, if the rate of discount is 9 % ?

3. Find the face of a 60-da. note which will yield \$ 250 when discounted at 7 %.

4. What was the face of the note at 90 da., on which I obtained \$ 875 in cash, after it was discounted at 6 % ?

5. What must be the face of a note dated June 28, and drawn at 2 mo., which will produce \$ 450 when discounted at $4\frac{1}{2}$ % ?

6. Find the sum for which a note at 2 mo. must be drawn so as to produce \$ 10,000 when discounted at 8 %.

7. I received \$ 1486.67 as proceeds of a note discounted for 64 da. at 5 %. What was the face of the note ?

8. A merchant bought goods for \$ 500 on terms 4/10, n/90. In order to take advantage of the discount offered for payment within 10 da., he borrowed \$ 500 from the bank on his 90-da. note. For how much was the note drawn, the rate of discount being 6 % ? How much did he gain by paying his bill in this way ?

MULTIPLES AND POWERS

269. Compare these expressions :

1. $3 \times 5 = 15$.

2. $5 \times 5 = 25$.

3. $2 \times 2 \times 3 \times 5 = 60$.

4. $5 \times 5 \times 5 = 125$.

In the first expression, 15 is a multiple of 5, and of 3. In the second expression, 25 is a multiple of 5. In the third expression, 60 is a multiple of 2, of 3, and of 5. In the fourth expression, 125 is a multiple of 5.

A **multiple** of a number is the **product** obtained by multiplying that number by another number.

15 is a multiple of 5 and 3; 30 is a multiple of 6 and 5.

270. In the second expression, 25 is obtained by multiplying 5 by **itself**, or by taking 5 as a factor twice. In the fourth expression, 125 is obtained by multiplying 5 by itself twice, or by taking 5 as a factor three times.

25 and 125 are called **powers** of 5.

A **power** of a number is the **product** obtained by multiplying that number by itself a certain number of times.

271. Since $2 \times 2 = 4$, 4 is the **second power** of 2.

$2 \times 2 \times 2 = 8$, 8 is the **third power** of 2.

$4 \times 4 = 16$, 16 is the **second power** of 4.

$4 \times 4 \times 4 = 64$, 64 is the **third power** of 4.

The second power of a number is called the **square** of the number.

The third power of a number is called the **cube** of the number.

We also speak of the 4th power, 10th power, etc. Powers are named from the number of times a number is taken as a factor.

272. Difference between Multiples and Powers

<p>A Multiple is the product obtained by multiplying a number by any other number.</p>	<p>Power is the name given to those products only that arise from the multiplication of a number by itself, a certain number of times.</p>
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273. The power to which a number is to be raised is indicated by a small figure called the **exponent**.

$(3)^2 = 3 \times 3 = 9$. The small 2 is the exponent.

$(2)^5 = 2 \times 2 \times 2 \times 2 \times 2$. The small 5 is the exponent.

274. To raise a fraction to a given power, raise both the numerator and the denominator to the given power.

$$\left(\frac{1}{2}\right)^2 = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}.$$

$$\left(\frac{3}{4}\right)^2 = \frac{(3)^2}{(4)^2} = \frac{9}{16}.$$

275. Any power of 1 is 1, because 1×1 any number of times always equals 1.

Every number is considered to be the first power of itself; the exponent 1 is not written.

276. To **square** a number is to raise it to the second power; 4 is the square of 2.

To **cube** a number is to raise it to the third power; 8 is the cube of 2.

EXERCISES

- 277.** 1. Square 2; 3; 5; 8; 4; 10; 12
 2. Cube 2; 3; 5; 6; 4; 1; 7

EXERCISES

278. Raise each number to the power indicated by the exponent.

- | | | | |
|--------------|-----------|---------------|-----------|
| 1. $2^2 = ?$ | $2^3 = ?$ | 6. $5^2 = ?$ | $5^3 = ?$ |
| 2. $2^4 = ?$ | $2^5 = ?$ | 7. $6^2 = ?$ | $6^3 = ?$ |
| 3. $3^2 = ?$ | $3^3 = ?$ | 8. $7^2 = ?$ | $7^3 = ?$ |
| 4. $3^4 = ?$ | $3^5 = ?$ | 9. $8^2 = ?$ | $8^3 = ?$ |
| 5. $4^2 = ?$ | $4^3 = ?$ | 10. $9^2 = ?$ | $9^3 = ?$ |

EXERCISES

279. Find values for :

Number	Second Power	Third Power	Number	Second Power	Third Power
1. 9	?	?	6. 30	?	?
2. 12	?	?	7. 50	?	?
3. 15	?	?	8. 10	?	?
4. 20	?	?	9. 100	?	?
5. 25	?	?	10. 1000	?	?

The second power of a number is called the **square** of the number.

The third power of a number is called the **cube** of the number.

We also speak of the 4th power, 10th power, etc. Powers are named from the number of times a number is taken as a factor.

272. Difference between Multiples and Powers

<p>A Multiple is the product obtained by multiplying a number by any other number.</p>	<p>Power is the name given to those products only that arise from the multiplication of a number by itself, a certain number of times.</p>
---	---

273. The power to which a number is to be raised is indicated by a small figure called the **exponent**.

$(3)^2 = 3 \times 3 = 9$. The small 2 is the exponent.

$(2)^5 = 2 \times 2 \times 2 \times 2 \times 2$. The small 5 is the exponent.

274. To raise a fraction to a given power, raise both the numerator and the denominator to the given power.

$$\left(\frac{1}{2}\right)^2 = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}.$$

$$\left(\frac{3}{4}\right)^2 = \frac{(3)^2}{(4)^2} = \frac{9}{16}.$$

275. Any power of 1 is 1, because 1×1 any number of times always equals 1.

Every number is considered to be the first power of itself; the exponent 1 is not written.

276. To **square** a number is to raise it to the second power; 4 is the square of 2.

To **cube** a number is to raise it to the third power; 8 is the cube of 2.

EXERCISES

- 277.** 1. Square 2; 3; 5; 8; 4; 10; 12
 2. Cube 2; 3; 5; 6; 4; 1; 7

EXERCISES

278. Raise each number to the power indicated by the exponent.

- | | | | |
|--------------|-----------|---------------|-----------|
| 1. $2^2 = ?$ | $2^8 = ?$ | 6. $5^2 = ?$ | $5^8 = ?$ |
| 2. $2^4 = ?$ | $2^5 = ?$ | 7. $6^2 = ?$ | $6^8 = ?$ |
| 3. $3^2 = ?$ | $3^3 = ?$ | 8. $7^2 = ?$ | $7^8 = ?$ |
| 4. $3^4 = ?$ | $3^5 = ?$ | 9. $8^2 = ?$ | $8^8 = ?$ |
| 5. $4^2 = ?$ | $4^3 = ?$ | 10. $9^2 = ?$ | $9^3 = ?$ |

EXERCISES

279. Find values for:

Number	Second Power	Third Power	Number	Second Power	Third Power
1. 9	?	?	6. 30	?	?
2. 12	?	?	7. 50	?	?
3. 15	?	?	8. 10	?	?
4. 20	?	?	9. 100	?	?
5. 25	?	?	10. 1000	?	?

EXERCISES

280. Raise the following fractions to the power indicated by the exponents:

1. $(\frac{1}{2})^2 = ?$ $(\frac{1}{2})^3 = ?$ 11. $(\frac{5}{6})^2 = ?$ $(\frac{5}{6})^3 = ?$
2. $(\frac{1}{2})^4 = ?$ $(\frac{1}{2})^5 = ?$ 12. $(\frac{3}{5})^2 = ?$ $(\frac{3}{5})^3 = ?$
3. $(\frac{1}{3})^2 = ?$ $(\frac{1}{3})^3 = ?$ 13. $(\frac{4}{5})^2 = ?$ $(\frac{4}{5})^3 = ?$
4. $(\frac{1}{3})^4 = ?$ $(\frac{1}{3})^5 = ?$ 14. $(\frac{2}{5})^2 = ?$ $(\frac{2}{5})^3 = ?$
5. $(\frac{1}{4})^2 = ?$ $(\frac{1}{4})^3 = ?$ 15. $(\frac{9}{10})^2 = ?$ $(\frac{9}{10})^3 = ?$
6. $(\frac{2}{3})^2 = ?$ $(\frac{2}{3})^3 = ?$ 16. $(\frac{3}{20})^2 = ?$ $(\frac{3}{20})^3 = ?$
7. $(\frac{2}{3})^4 = ?$ $(\frac{2}{3})^5 = ?$ 17. $(\frac{7}{12})^2 = ?$ $(\frac{7}{12})^3 = ?$
8. $(\frac{3}{4})^2 = ?$ $(\frac{3}{4})^3 = ?$ 18. $(\frac{3}{100})^2 = ?$ $(\frac{3}{100})^3 = ?$
9. $(\frac{3}{4})^4 = ?$ $(\frac{3}{4})^5 = ?$ 19. $(\frac{2}{25})^2 = ?$ $(\frac{2}{25})^3 = ?$
1. $(\frac{3}{8})^2 = ?$ $(\frac{3}{8})^3 = ?$ 20. $(\frac{7}{30})^2 = ?$ $(\frac{7}{30})^3 = ?$

EXERCISES

281. Raise the following to the powers indicated:

1. $(.5)^2 = ?$ $(.5)^3 = ?$ 6. $(1.5)^2 = ?$ $(1.5)^3 = ?$
2. $(.05)^2 = ?$ $(.05)^3 = ?$ 7. $(2.5)^2 = ?$ $(2.5)^3 = ?$
3. $(.10)^2 = ?$ $(.10)^3 = ?$ 8. $(.03)^2 = ?$ $(.03)^3 = ?$
4. $(.25)^2 = ?$ $(.25)^3 = ?$ 9. $(1.1)^2 = ?$ $(1.1)^3 = ?$
5. $(.01)^2 = ?$ $(.01)^3 = ?$ 10. $(7.5)^2 = ?$ $(7.5)^3 = ?$

EXERCISES

282. Raise the following to the powers indicated:

1. $(1\frac{1}{2})^2 = ?$ $(\frac{3}{2})^2 = ?$ 6. $(1\frac{1}{10})^2 = ?$ $(1\frac{1}{10})^3 = ?$
2. $(2\frac{1}{2})^2 = ?$ $(2\frac{1}{2})^3 = ?$ 7. $(2\frac{1}{5})^2 = ?$ $(2\frac{1}{5})^3 = ?$
3. $(1\frac{1}{4})^2 = ?$ $(1\frac{1}{4})^3 = ?$ 8. $(1\frac{1}{8})^2 = ?$ $(1\frac{1}{8})^3 = ?$
4. $(1\frac{1}{3})^2 = ?$ $(1\frac{1}{3})^3 = ?$ 9. $(3\frac{1}{2})^2 = ?$ $(3\frac{1}{2})^3 = ?$
5. $(2\frac{1}{3})^2 = ?$ $(2\frac{1}{3})^3 = ?$ 10. $(2\frac{1}{4})^2 = ?$ $(2\frac{1}{4})^3 = ?$

FACTORS AND ROOTS

283. Compare these expressions :

1. $15 = 3 \times 5$.

2. $25 = 5 \times 5$.

3. $60 = 2 \times 2 \times 3 \times 5$.

4. $125 = 5 \times 5 \times 5$.

In the first expression, 3 and 5 are factors of 15.

In the second expression, 5 is a factor of 25.

In the third expression, 2, 3, and 5 are factors of 60. In the fourth expression, 5 is a factor of 125.

A **factor** of a number is **one** of the **whole numbers** which, when multiplied by another number, produces that number. 3 is a factor of 15, because 3 multiplied by 5 = 15.

Every exact divisor of a number is a factor of that number. The quotient obtained by dividing a number by one factor is the other factor; 2 is a factor of 18; $18 \div 2 = 9$; 9 is the second factor of 18. (See also p. 49.)

284. In the second expression, the two factors of 25 are equal. In the fourth expression, the three factors of 125 are equal.

Therefore, 5 is said to be a **root** of 25, and 5 is a **root** of 125.

A **root** of a **number** is **one** of the **equal factors** which, when multiplied together, produce the number.

285. Since $2 \times 2 = 4$, 2 is called the **square root** of 4.

$2 \times 2 \times 2 = 8$, 2 is called the **cube root** of 8.

The **square root** of a number is **one** of the two **equal factors** of the number.

The **cube root** of a number is **one** of the **three equal factors** of the number.

We also speak of the fourth root of a number; the fifth root, etc. Roots are named from the number of equal factors necessary to produce the number.

$2 \times 2 \times 2 \times 2 \times 2 = 32$; therefore, 2 is the ? root of 32.

Difference between Factors and Roots

286. A factor is **any one** of the whole numbers which, when multiplied, produce a number.

A **root** of a number is **one** of the equal factors of a number.

The root which is to be taken is indicated by a **root sign** or **radical sign**, $\sqrt{\quad}$, and a small figure called the **index** of the root.

$\sqrt[3]{8}$ means that the cube root of 8 is to be taken; 3 is the index of the root.

$\sqrt[5]{32}$ means that the fifth root of 32 is to be taken; 5 is the index of the root.

When the square root is to be taken, the index 2 is usually omitted; $\sqrt{4}$ means the square root of 4.

287. Any root of 1 is 1, because 1 multiplied by itself any number of times always equals 1.

Every number is considered to be the first root of itself.

288. Proof. To prove the work, raise the root to the required power; if the original number is obtained, the answer is correct.

EXERCISES

289. Find the roots indicated. (Prove those marked *.)

- | | | |
|------------------|----------------------|----------------------|
| 1. $\sqrt{9}$ | 6. $\sqrt{16}$ | 11. $\sqrt[3]{8}$ |
| 2. $\sqrt{100}$ | * 7. $\sqrt{36}$ | 12. $\sqrt[3]{125}$ |
| * 3. $\sqrt{25}$ | 8. $\sqrt{64}$ | 13. $\sqrt[4]{16}$ |
| 4. $\sqrt{49}$ | 9. $\sqrt[3]{27}$ | 14. $\sqrt{1}$ |
| 5. $\sqrt{81}$ | * 10. $\sqrt[3]{64}$ | 15. $\sqrt[3]{1000}$ |

EXERCISES

290. Find the roots indicated. (Prove those marked *.)

1. $\sqrt[4]{256}$

Process

Factoring, $\sqrt[4]{256} = \sqrt[4]{4 \times 4 \times 4 \times 4} = 4$. *Ans.*

- | | | |
|----------------------|----------------------|------------------------|
| 2. $\sqrt{625}$ | 6. $\sqrt[4]{81}$ | 10. $\sqrt[5]{10,000}$ |
| 3. $\sqrt[4]{625}$ | * 7. $\sqrt{400}$ | 11. $\sqrt[6]{729}$ |
| 4. $\sqrt[4]{1296}$ | 8. $\sqrt[3]{1000}$ | * 12. $\sqrt[3]{512}$ |
| * 5. $\sqrt[5]{243}$ | * 9. $\sqrt{10,000}$ | 13. $\sqrt{3725}$ |

EXERCISES

- | | | |
|----------------------------------|-----------------------------------|----------------------------------|
| 1. $\sqrt{\frac{25}{36}} = ?$ | 4. $\sqrt[3]{\frac{27}{64}} = ?$ | 7. $\sqrt{\frac{64}{729}} = ?$ |
| 2. $\sqrt[3]{\frac{1}{8}} = ?$ | 5. $\sqrt[3]{\frac{8}{125}} = ?$ | * 8. $\sqrt{\frac{25}{169}} = ?$ |
| * 3. $\sqrt{\frac{49}{100}} = ?$ | * 6. $\sqrt{\frac{100}{144}} = ?$ | 9. $\sqrt{\frac{625}{16}} = ?$ |

MENSURATION

LINES AND ANGLES

291. An **angle** is the **difference of direction** of two lines that proceed from the same point.

The point A is called the **vertex** of the angle.



FIG. 1.

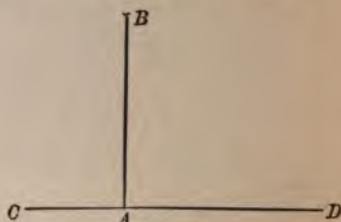


FIG. 2.

292. The angles in Fig. 2 are called **right angles**.

Right angles are sometimes called **square corners**.

The lines AB and CD are said to be **perpendicular** to each other.

PLANE SURFACES: RECTANGLES

293. A **rectangle** is a plane figure bounded by four straight lines and having four right angles.

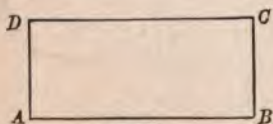


FIG. 3.

AB (or CD) is the **base** of the rectangle.

AD (or BC) is the **altitude** of the rectangle.

294. The **area** of a **rectangle** is the surface enclosed by the four sides of the rectangle.

The unit of measurement of small surfaces or areas is the **inch square**. An inch square is a square each side of which is 1 in. long. The surface or area of an inch square is 1 sq. in. Larger areas or surfaces are measured in square feet or square yards, etc.

295. RULE. The area of a rectangle equals the product of the base by the altitude, expressed in like units.

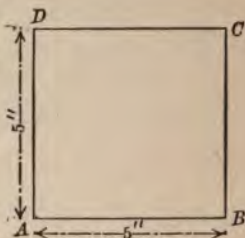
EQUATION AND FORMULA. Area Rectangle = $b \times a$.

To find the Area of a Square

296. A square is a rectangle in which the four sides are equal.

297. EQUATION AND FORMULA.

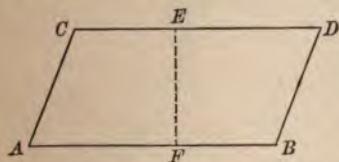
Area Square = $b \times b = b^2$.



NOTE. $b^2 = b \times b$. The small 2 is called an exponent. It means that b is multiplied by itself, or that b is taken twice as a factor; e.g. $5^2 = 5 \times 5 = 25$.

PARALLELOGRAMS

298. The figure $ABCD$ is a **parallelogram** because the lines AB and CD are parallel (they never meet) and the lines AC and BD are parallel. AB is the **base**, and EF is the **altitude** of the parallelogram.



Parallelogram. AB and CD are parallel. AC and BD are parallel. EF is the altitude.

299. RULE. The area of a parallelogram equals the product of the base by the altitude, expressed in like units.

EQUATION AND FORMULA.

$$\text{Area Parallelogram} = b \times a.$$

TRIANGLES

300. These 3 figures are triangles.

A **triangle** is a plane figure bounded by 3 straight lines.

AB is the **base** of the triangle; CD is the **altitude** or height of the triangles in Figs. 1 and 3.

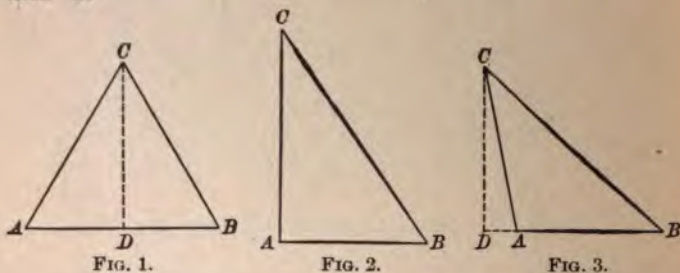


Fig. 2 represents a right triangle; it contains a right angle at A ; in Fig. 2, CA is the altitude or height.

The altitude is always perpendicular to the base (or to the base prolonged).

301. RULE. The area of a triangle equals one half of the product of the base by the altitude, expressed in like units.

EQUATION AND FORMULA.

$$\text{Area Triangle} = \frac{1}{2} b \times a, \text{ or } \frac{b \times a}{2}.$$

EXERCISES

302. Find the area of the following rectangles:

	Length	Width		Length	Width
1.	5 ft.	3 ft.	4.	$10\frac{1}{2}$ yd.	$3\frac{1}{2}$ ft.
2.	$3\frac{1}{2}$ ft.	$2\frac{1}{2}$ ft.	5.	6 ft.	9 in.
3.	9 ft. 6 in.	2 ft. 4 in.	6.	12.7 ft.	3.5 yd.

Of each of the following parallelograms:

	Base	Altitude		Base	Altitude
7.	22.7 ft.	16.5 ft.	9.	300 ft.	6 ft. 2 in.
8.	12 ft. 9 in.	5 ft. 4 in.	10.	500 ft.	9.3 yd.

PROBLEMS

303. 1. The width of a rectangle is 40 ft.; the length is $2\frac{1}{2}$ times the width. Find the area.

2. The base of a rectangle is 50 ft.; the altitude is $9\frac{1}{2}$ ft. Find the area.

3. The base of one rectangle is 50 ft.; the altitude is $9\frac{1}{2}$ ft.; the base and altitude of another rectangle are twice as great as the base and altitude of the first rectangle. Find the area of the second rectangle. The area of the second rectangle is how many times the area of the first rectangle?

4. The area of a rectangle is 750 sq. yd.; the base is 50 yd. Find the altitude.

5. The area of a rectangle is 400 sq. ft. The base is 4 times the altitude. Find the base and the altitude.

EXERCISES

304. Find the area of each of the following triangles :

	Base	Altitude		Base	Altitude
1.	10 in.	6 in.	4.	30 ft. 6 in.	5 ft. 2 in.
2.	$4\frac{1}{2}$ yd.	$2\frac{1}{2}$ ft.	5.	35.6 ft.	4.5 in.
3.	20 ft.	4 in.	6.	52.7 ft.	3.4 in.

PROBLEMS

305. 1. The base of a triangle is 20 ft. ; the altitude is $\frac{1}{2}$ the base. Find the area.

2. The base of a triangle is 18 in. ; the altitude is $3\frac{1}{2}$ times the base. Find the area.

3. Find the area of a triangle if the base equals 11.5 ft. and the altitude is twice as great as the base.

4. The area of a triangle equals 400 sq. ft. The base is 10 ft. Find the altitude.

5. The area of a triangle is 225 sq. ft. The base is twice as long as the altitude. Find the dimensions of the triangle.

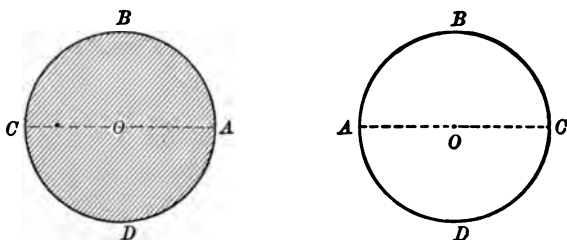
THE CIRCLE

306. The shaded surface *ABCD* is a **circle**.

The curved line *ABCD* is the **circumference** of the circle.

The point *O* is the **center** of the circle. The line *OA* is the **radius** of the circle. The line *AC* is the **diameter** of the circle. The diameter is twice as long as the radius.

307. A **circle** is a plane surface, bounded by a curved line, every point of which is equally distant from a point within, called the center.



To find the Relation between the Circumference and the Diameter of a Circle

308. Draw on cardboard a circle having a radius of $3\frac{1}{2}$ inches; then cut out the circle; place a string carefully along the circumference. Measure the length of the string; it is 11 inches. Draw other circles with different diameters. Make a table showing the lengths of the diameters and circumferences as follows:

	DIAMETER	CIRCUMFERENCE	RATIO OF CIRCUMFERENCE TO DIAMETER (CIRCUMFERENCE ÷ DIAMETER)
1.	$3\frac{1}{2}$ in.	11 in.	?
2.	—	—	—
etc.	etc.	etc.	etc.

It will be found that in each circle the circumference is about $3\frac{1}{7}$ times as long as the diameter. If we measured the length of the circumference

more accurately, it would be found that the circumference is 3.1416 times the diameter.

309. RULE. The Circumference of a circle equals $3\frac{1}{7}$ times the diameter.

FORMULA. $C = 3\frac{1}{7} \times d$,

or, more accurately, $C = 3.1416 \times d$.

The Greek letter π (π) is used to denote $3\frac{1}{7}$, or 3.1416. r is used to denote the radius of the circle.

310. Therefore, $C = \pi \times d$,

or $C = 2 \times \pi \times r$

(because the diameter equals twice the radius).

To find the Area of a Circle



FIG. 1.

311. Draw a circle having a radius of $3\frac{1}{2}$ in. Divide it into 16 portions (called sectors) as shown in Fig. 1.

Cut out the sectors and arrange them as in Figs. 2 or 3.

The sectors form a figure that is almost a parallelogram. The base AB is nearly $\frac{1}{2}$ the circumference; the height of the figure is



FIG. 2.

nearly equal to the radius of the circle. If the circle be divided into an infinitely large number of

sectors, the figure $ABCD$ tends to become a parallelogram, having a base equal to $\frac{1}{2}$ the circumference and an altitude equal to the radius.



FIG. 3.

Area of parallelogram = base \times altitude.

Area of circle = $\frac{1}{2}$ (circumference \times radius),

or $\frac{1}{2}(2 \times \pi \times r \times r) =$

The area of a circle equals $\frac{1}{2}$ of the product of the circumference by the radius.

312. A second rule is obtained from the first as follows :

Substitute $2 \times \pi \times r$ for circumference (see Art. 310), and the formula becomes

$$\text{Area of Circle} = \frac{1}{2} \times 2 \times \pi \times r \times r,$$

or

$$\text{Area of Circle} = \pi \times r^2.$$

RULE. The area of a circle equals 3.1416 * times the square of the radius.

EXERCISES

313. Find the area of a circle whose diameter is :

- | | | |
|-----------------------|-----------|----------------|
| 1. 6 ft. | 3. 3 m. | 5. 1 ft. 2 in. |
| 2. $2\frac{1}{2}$ in. | 4. 15 cm. | 6. 2.8 ft. |

* NOTE. It is sufficiently accurate to consider $\pi = 3\frac{1}{4}$.

Find the circumference of a circle whose radius is:

- | | | |
|-----------------------|-----------------------|-----------------|
| 7. 3 in. | 9. $4\frac{3}{4}$ in. | 11. 70 cm. |
| 8. $3\frac{1}{2}$ ft. | 10. 5 m. | 12. 4 ft. 2 in. |

Find the area of a circle whose radius is:

- | | | |
|------------|-------------------------|-----------------------|
| 13. 22 in. | 15. 24.7 ft. | 17. 19.3 yd. |
| 14. 2 m. | 16. $28\frac{1}{2}$ ft. | 18. $3\frac{1}{2}$ m. |

PROBLEMS

314. 1. A circular plot of ground is 28 ft. in diameter. Find its value at \$ 3 per sq. ft.

2. The diameter of the inner rim of a wheel is 30 in. How long is the rim?

3. What is the length of the circumference of a circle described by a compass whose points are 12 in. apart?

4. The distance around a circular plot of ground is 440 ft. What is the greatest distance across the plot?

5. (a) The diameter of a wheel of a bicycle is 26 in. How far does the wheel go in making one revolution?

(b) How many revolutions are necessary to go $\frac{1}{2}$ mi.?

6. A circular track has a diameter of 350 ft. A boy rides around the track 3 times. How far has he traveled?

7. How many times must the boy ride around the track mentioned in No. 6, to travel 1 mi.?

8. One circle has a radius of 9 inches. A second circle has a radius twice as great. How much greater is the area of the second circle than the area of the first circle?

9. How much greater is the circumference of the second circle mentioned in No. 8 than the circumference of the first circle?

10. How many square feet of sheet iron are required to make a cover for the top of a circular tank that is 6 ft. in diameter?

11. How long are the spokes of a wheel that has an inner circumference of 56 in.?

12. How long are the spokes of a wheel the inner circumference of which is twice as great as the circumference given in No. 11?

PRACTICAL APPLICATIONS

CARPETING

315. Carpet is sold in strips, usually $\frac{3}{4}$ yd. wide. In ascertaining the cost of carpet, it is necessary to know :

First : Whether the carpet is to be laid along the length or along the width of the room, because the number of strips may vary according to the way the carpet is laid.

Second : The total number of yards; this is found by multiplying the number of yards in each strip by the number of strips.

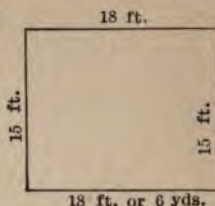
An additional strip may have to be purchased when the dimensions of a floor are not exact multiples of the width of a strip. For example, if a room is 18 ft. wide, 8 strips of carpet, $\frac{3}{4}$ yd. wide, are sufficient; but if the room is 19 ft. wide, the number of strips needed is $8\frac{1}{3}$; therefore, 9 strips must be purchased.

Some carpet may be wasted in matching patterns. The purchaser must pay for portions of a strip not used and for pieces wasted in matching.

Carpet is sold only in yards, half yards, quarter yards, and eighth yards.

To find the Number of Yards and the Cost of Carpet

316. 1. How many yards of carpet, $\frac{3}{4}$ yd. wide, are required for a floor, 18 ft. long, and 15 ft. wide, if the carpet is laid along the length of the room?



Process

Number of strips $= 5 \div \frac{3}{4} = 5 \times \frac{4}{3} = 6\frac{2}{3}$.

Therefore, 7 strips must be purchased.

Total amount of carpet $= 7 \times 6$ yd. $= 42$ yd. *Ans.*

2. How many yards would be needed if the carpet were laid along the width of the room?

Process

Number of strips $= 6 \div \frac{3}{4} = 8$ (no remainder).

Total amount of carpet $= 8 \times 5$ yd. $= 40$ yd. *Ans.*

Therefore, 2 yd. less would be needed.

3. How many yards of carpet $\frac{3}{4}$ yd. wide are required for a floor, 18 ft. long and 21 ft. wide, if the carpet is laid along the length of the room? Allow for a loss of 6 in. for matching.

Process

Number of strips $= 6 \div \frac{3}{4} = 8$.

Each strip is 7 yd. + 3 in. long* $= 7\frac{1}{2}$ yd.

Total amount of carpet $= 8 \times 7\frac{1}{2} = 56\frac{3}{4}$, or $56\frac{3}{4}$ yd. *Ans.*

* NOTE. Allowance is made here for loss in matching on each strip; but there is no loss on the first strip.

PROBLEMS

317. Find the cost of carpet for each of the following. Pupils are to decide whether the carpet should be laid along the length or the width of the room.

DIMENSIONS OF FLOOR			WIDTH OF CARPET	LOSS PER STRIP IN MATCHING	COST PER YARD
Length		Width			
1.	18 ft.	21 ft.	1 yd.	—	\$ 1.25
2.	15 ft.	18 ft.	$\frac{3}{4}$ yd.	—	\$ 1.25
3.	21 ft.	27 ft.	$\frac{3}{4}$ yd.	3 in.	\$ 2.25
4.	30 ft.	30 ft.	$\frac{3}{4}$ yd.	—	\$ 2.75
5.	27 ft.	24 ft.	$\frac{3}{4}$ yd.	6 in.	\$ 3.00
6.	100 ft.	9 ft.	$\frac{3}{4}$ yd.	—	\$ 1.50
7.	60 ft.	9 ft.	$\frac{3}{4}$ yd.	—	\$ 1.75

8. How much is saved by covering a floor which is 24 ft. by 27 ft. with oilcloth at 50 ct. per square yard rather than with carpet $\frac{3}{4}$ yd. wide at \$1.50 per yard?

PAPERING

318. Wall paper is sold in rolls 18 in. wide; a single roll usually contains 8 yd., a double roll 16 yd.

An entire roll must be paid for, if any portion of it is used.

Since wall paper is 18 in. wide ($\frac{1}{2}$ yd. wide), the

number of strips equals twice the perimeter of the floor of the room expressed in yards.

Allowance may be made for doors, windows, borders, bases, loss in matching, etc.

To find the Cost of Papering a Room

319. 1. A room is 18 ft. long, 15 ft. wide, 15 ft. high. How many double rolls of paper 18 in. wide are required? (Make no allowance for doors, windows, etc.)

Process

Perimeter of floor = $(2 \times 18 \text{ ft.}) + (2 \times 15 \text{ ft.}) = 66 \text{ ft.} = 22 \text{ yd.}$

Number of strips = $2 \times 22 = 44$.

A double roll is 48 ft. long.

Number of strips from a double roll = $48 \div 15 = 3\frac{1}{5}$.

Number of double rolls = $44 \div 3\frac{1}{5} = 13\frac{3}{4}$, or 14 rolls. *Ans.*

2. How many double rolls of paper are required for the walls of a room 21 ft. long, 18 ft. wide, 15 ft. high, allowing for a door $3\frac{1}{2}$ ft. wide, 2 windows, each $3\frac{1}{2}$ ft. wide, a border 1 ft. wide, and a base 1 ft. high?

Process

Perimeter of floor = $(2 \times 21 \text{ ft.}) + (2 \times 18 \text{ ft.}) = 78 \text{ ft.}$

Allowance, doors and windows $\frac{10\frac{1}{2}}{2} \text{ ft.}$

Remainder $\frac{67\frac{1}{2}}{2} \text{ ft.,}$
or $22\frac{1}{2} \text{ yd.}$

Number of strips = $2 \times 22\frac{1}{2} = 45$.

15 ft. — 1 ft. — 1 ft. = 13 ft. = length of each strip.

Number of strips from a double roll = $48 \div 13 = 3\frac{9}{13}$.

Number of double rolls = $45 \div 3\frac{9}{13} = 12\frac{3}{16} = 13$ double rolls.

Ans.

(Or 12 double rolls and 1 single roll if this paper can be bought in single rolls.)

PROBLEMS

320. Find the cost of papering the walls of each of the following:

1. A room 24 ft. long, 18 ft. wide, 15 ft. high. Allow for a base 1 ft. high, and a border 1 ft. wide. Use double rolls at \$ 1 per roll.

2. Use the dimensions in problem 1. Allow for the same base and border, and also for 2 doors, each $3\frac{1}{2}$ ft. wide, and 2 windows, each $3\frac{1}{2}$ ft. wide. Use double rolls at \$ 1.25 per roll.

3. Use the dimensions in problem 1. Allow for a base 1 ft. high ; the paper is to be laid 8 ft. high. Make no other allowances. Use double rolls at \$ 1.50 per roll.

4. A hall 60 ft. long, 8 ft. wide, 20 ft. high. The double doors at the front of the hall extend along the entire width from the floor to the ceiling. Allow also for a border 1 ft. wide. Use double rolls at \$ 1.50 per roll.

5. A bathroom 12 ft. long, 8 ft. wide, and 15 ft. high. The door at the front of the room extends along half the width ; a window at the other end is half the width of the room. The door and window extend from the floor to the ceiling. The lower half of the room is tiled ; the upper half is to be papered. Use single rolls at 90 ct. a roll.

PLASTERING AND PAINTING

321. The cost of plastering and painting is based on the cost of a square yard. Allowances are sometimes made for doors, windows, etc., but there is no uniform custom.

To find the Cost of Plastering and Painting

PROBLEMS

1. Find the cost of plastering the walls and ceiling of a room 24 ft. long, 18 ft. wide, 15 ft. high. Allow for a baseboard 1 ft. high. Cost, \$ 1.25 per square yard.

2. Find the cost of painting the walls of the room, including the baseboard, at 22 ct. per square yard.

3. Find the cost of kalsomining the ceiling of the room at 11 ¢ per square yard.

4. The walls and ceiling of a room are $20\frac{1}{2}$ by 18 by $16\frac{1}{2}$ ft. Allow for a door $3\frac{1}{2}$ ft. wide and $16\frac{1}{2}$ ft. high and two windows, each $3\frac{1}{2}$ ft. wide and $16\frac{1}{2}$ ft. high. Find the cost of plastering at \$ 1.25 per square yard.

5. Find the cost of plastering both sides and the ceiling of a hall 60 ft. long, 8 ft. wide, and 22 ft. high, at \$ 1.10 per square yard. (Doors at both ends extend all the way across the width of the hall from the floor to the ceiling.)

6. The walls and ceiling of a bathroom are 12 ft. long, 8 ft. wide, and 15 ft. high. Allow for a door at one end, and a window at the other, each extending halfway across the width from the floor to the ceiling. The lower half of the room is tiled. Find the cost of plastering the ceiling and the upper half of the walls, at \$ 1.15 per square yard.

7. Find the cost of varnishing both sides of the door in No. 4, at 85 ct. per square yard.

8. Find the cost of painting the walls of the hall in No. 5, at 22 ct. per square yard, and of varnishing both sides of the doors, at 85 ct. per square yard.

9. Find the cost of papering the ceiling of the room in No. 6, with single rolls $\frac{3}{4}$ yd. wide, at 75 ct. per roll.

10. Find the cost of tiling the lower half of the walls in a bathroom 12 ft. long, 8 ft. wide, and 16 ft. high, allowing for a door 4 ft. by 16 ft., and a window 4 ft. by 16 ft., at 60 ct. per square foot.

LUMBER MEASURE OR BOARD MEASURE

322. The unit of measurement of lumber is the **board foot**; it is the amount of lumber in a board 1 ft. long, 1 ft. wide, and 1 in. thick.

A board 18 ft. long, 1 ft. wide, 1 in. thick contains 18 board feet; a board 12 feet long, 1 ft. wide, $2\frac{1}{2}$ in. thick contains 30 board feet ($12 \times 1 \times 2\frac{1}{2} = 30$).

323. To find the number of board feet:

RULE. Find the product of the length in feet, the width in feet, and the thickness in inches; the result is the number of board feet.

In writing, the sign ' is used for feet, and " for inches. In finding the number of board feet, the width is reckoned to the nearest half inch; a board $8\frac{1}{4}$ in. wide is reckoned 8 in. wide, a board $8\frac{5}{8}$ in. wide is reckoned $8\frac{1}{2}$ in. wide.

EXERCISES**324.** Find the number of board feet in :

1. A board 1" thick, 9" wide, 13' long.
2. A board 1" thick, $10\frac{1}{4}$ " wide, 8' long.
3. A board 2" thick, 14" wide, 10' long.
4. A board $1\frac{1}{2}$ " thick, $8\frac{1}{2}$ " wide, 15' long.
5. A board $2\frac{1}{4}$ " thick, $11\frac{1}{2}$ " wide, $12\frac{1}{2}$ ' long.
6. 60 pieces, each $1'' \times 8'' \times 13'$.
7. 50 pieces, each $1\frac{1}{4}'' \times 9\frac{5}{8}'' \times 12'$.
8. 60 pieces, each $2'' \times 7\frac{5}{8}'' \times 14'$.
9. 35 pieces, each $3'' \times 10\frac{1}{4}'' \times 15'$.
10. 90 pieces, each $\frac{3}{4}'' \times 9\frac{1}{4}'' \times 15'$.
11. 70 pieces, each $1'' \times 15\frac{5}{8}'' \times 19\frac{1}{2}'$.
12. 75 pieces, each $1\frac{1}{2}'' \times 10\frac{5}{8}'' \times 14\frac{1}{2}'$.
13. 120 pieces, each $1\frac{1}{4}'' \times 12\frac{1}{4}'' \times 10'$.

Find the cost of:

14. 120 pieces, each $1\frac{1}{2}$ " thick, $11\frac{1}{2}$ " wide, 15' long, at 7 ct. per ft.

15. 90 pieces, each $2\frac{1}{4}$ " \times $9\frac{5}{8}$ " \times $14\frac{1}{2}$ ', at $7\frac{1}{2}$ ct. per ft.

16. 325 pieces, each 1 " \times $14\frac{3}{4}$ " \times 17', at $14\frac{1}{2}$ ct. per ft.

17. 250 pieces, each $1\frac{1}{4}$ " \times $13\frac{1}{4}$ " \times $19\frac{1}{2}$ ', at $8\frac{1}{2}$ ct. per ft.

ROOFING AND FLOORING

325. The cost of roofing and flooring is usually based on the cost of a **square** 10 ft. long, or **100 sq. ft.**

Shingles are usually 16 in. or 18 in. long, and 4 in. wide. They are laid 4 in. or $4\frac{1}{2}$ in. "to the weather," each shingle covering about 18 sq. in. of roof.

Therefore, about **800 shingles** are required to cover a **square** (100 ft.), but in order to allow for waste, etc., from 900 to 1000 shingles are allowed for each 100 sq. ft. of roof surface.

Shingles are sold in **bundles** containing **250**. Dealers do not sell less than a bundle.

In laying floors, the "tongue" of one board fits into the "groove" of the next board. There is, therefore, a loss in the surface covered by the board. There is always some waste in cutting and fitting lumber.

To find the Cost of Roofing and Flooring

PROBLEMS

326. 1. Find the cost of laying the floor of a room 20 ft. long and 18 ft. wide, at \$3.50 per thousand square feet.

2. How many bundles of shingles are needed to cover a gable roof 30 ft. long and 20 ft. wide, allowing 1000 shingles to the square?

3. Find the cost of the shingles for No. 2 at \$3.85 per thousand.

4. Find the cost of flooring a room 24 ft. by 20 ft., at \$3.20 per thousand square feet, allowing for a loss in cutting and overlapping $\frac{1}{5}$ of the floor area.

5. Find the cost of shingling a gable roof 30 ft. by 20 ft. with slate shingles, at \$12.50 per square.

6. The flat roof of a barn is 60 ft. by 35 ft. How many bundles of shingles must be purchased to cover the roof, allowing 900 shingles to the square?

7. How much will the shingles cost at \$4 per M?

8. How long will it take a man to lay the shingles, if he can lay 2000 shingles a day?

9. Find the cost of laying a wooden floor and wooden ceiling of a room 24 ft. long and 22 ft. wide at \$4 per thousand square feet.

10. A hall is 100 ft. long and 8 ft. wide. Find the cost of laying a floor at \$2.90 per thousand square feet, allowing for a loss of $\frac{1}{6}$ of the floor area for waste.

11. How many square feet are covered by a board 22 ft. long and 10 in. wide?

12. If a board 18 ft. long and 9 in. wide and 3 in. thick is cut into boards 1 in. in thickness, how many square feet do they cover?

13. How many board feet in 570 cu. ft.?

14. How many feet of wood will be required for a fence 1 mi. long, 5 boards high, each board being 4 in. wide and 1 in. thick?

15. A banquet hall 60 ft. long and 35 ft. 8 in. wide is to be covered with maple flooring costing \$30 per 1000 ft. A man getting \$5 per day can lay 200 sq. ft. Find the cost of flooring the hall.

16. An apartment of 5 rooms is to be covered with yellow pine flooring costing \$20 per 1000 ft. The dimensions of the rooms are:

8 ft. 10 in. \times 7 ft. 6 in.

7 ft. 10 in. \times 7 ft.

12 ft. \times 10 ft. 9 in.

6 ft. 6 in. \times 5 ft. 6 in.

6 ft. 10 in. \times 5 ft. 4 in.

Find the total cost of the lumber.

17. How many board feet of lumber are in a pile 8 ft. high, 9 ft. wide, and 20 ft. long? Find the value of the lumber at \$15 per 1000 ft.

18. How many feet, board measure, in 22 boards, each board being 19 ft. long, 10 in. wide, and 1 in. thick?

19. How many board feet in 32 joists, each joist being 20 ft. long and 5 in. square?

20. How many feet of lumber will be required for a box 8 ft. high, 4 ft. square?

SOLIDS

FOR READING AND DISCUSSION

328. Anything that occupies space is a solid; *e.g.* a baseball, a book, a pencil, etc. Solids have three dimensions, namely, **length**, **breadth**, and **thickness**;

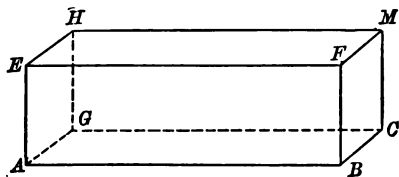


FIG. 1.

surfaces have only two dimensions, length and breadth; lines have only one dimension, length.

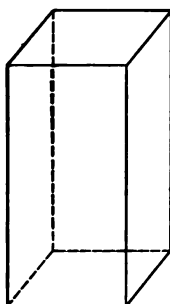
Fig. 1 is not a solid, but it represents a solid.

The lines AB , BC , AE , etc., represent the **edges** of a solid.

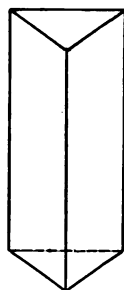
The surfaces $AEFB$, $BCMF$, $CMHG$, and $GHEA$ are the **lateral faces** of the solid.

329. A **rectangular solid** is a solid having 6 rectangular faces or surfaces. Fig. 1 represents a rectangular solid. The lower and upper faces ($ABCG$ and $EFMH$) are called the lower and upper **bases** of the solid.

330. A **prism** is a solid whose lateral faces are parallelograms, and whose upper and lower bases are equal and parallel to each other.



Rectangular Prism



Triangular Prism

Prisms are named from the shape of their bases — as triangular prisms, rectangular prisms, square prisms, hexagonal prisms, etc.

331. By the **lateral surface** of a prism is meant the total surface of all the faces except the upper and lower bases.



332. A **circular cylinder** is a solid bounded by a uniformly curved surface and having circles parallel to each other for its upper and lower bases.

333. The lateral surface of a cylinder is the curved surface.

334. The total surface of a prism or a cylinder consists of the lateral surface plus the upper and lower bases.

SURFACES OF SOLIDS

To find the Area of the Lateral Surface of a Prism

335. A rectangular solid is 4 ft. long, 2 ft. wide, and 3 ft. high. Find the lateral surface.

Process

The lateral surface consists of the areas of 4 rectangles.

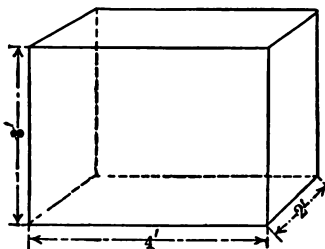
Area of one rectangle, 4 ft. by 3 ft. = 12 sq. ft.

Area of one rectangle, 4 ft. by 3 ft. = 12 sq. ft.

Area of one rectangle, 2 ft. by 3 ft. = 6 sq. ft.

Area of one rectangle, 2 ft. by 3 ft. = 6 sq. ft.

Lateral surface = 36 sq. ft. *Ans.*



A shorter method is to find the perimeter of the base (12 ft.) and multiply it by the height. The result, expressed in square units of the same denominations as the dimensions, is the lateral surface.

336. RULE. The area of the convex surface of a prism equals the perimeter of the base multiplied by the altitude, expressed in like units.

To find the Area of the Convex Surface of a
Circular Cylinder



FIG. 1

1. A cylinder is 6 ft. high; the diameter of the base is 2 ft. Find the convex surface.

Wrap a piece of paper around the cylinder so that it will equal the convex surface; then lay the paper as shown in Fig. 2. The paper will have the shape of a rectangle. The area of the rectangle, which is equal to the convex surface of the cylinder, is found by multiplying the length (the circumference of the base) by the altitude (the altitude of the cylinder).

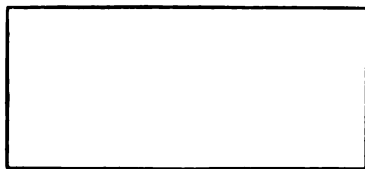


FIG. 2

Convex Surface of Circular Cylinder = $3.1416 \times 2 \times 6 = 37.6992$ (sq. ft.).

337. RULE. The area of the convex surface of a circular cylinder equals the circumference of the base multiplied by the altitude, expressed in like units.

Formula $CS = 2\pi \times r \times h$.

EXERCISES

338. Find the area of the lateral surface of the following rectangular prisms :

Base	Altitude
1. 3 ft. by 6 ft.	5 ft.
2. 2 ft. by 3 ft.	4 ft.
3. $2\frac{1}{2}$ m. by $3\frac{1}{2}$ m.	$2\frac{1}{2}$ m.
4. 9 dm. by 10 dm.	2 dm.

Find the area of the lateral surface of the following prisms :

5. Triangular prism ; perimeter of base, 18 ft. ; altitude, 3 ft.

6. Hexagonal prism ; perimeter of base, 12 ft. ; altitude, 3 ft.

Find the area of the lateral surface of the following circular cylinders :

7. Circumference of base, 21 in. ; altitude, 6 in.

8. Diameter of base, 2 ft. ; altitude, 1 ft.

9. Find the total surface of the circular cylinders in exercises 7 and 8.

PROBLEMS

339. 1. How much sheet iron is necessary to line the sides of a refrigerator 18 ft. long, 12 ft. wide, and 8 ft. high, adding 10 % for waste ?

2. Find the total surface of a cube 15 in. long.

10. A hall is 100 ft. long and 8 ft. wide. Find the cost of laying a floor at \$2.90 per thousand square feet, allowing for a loss of $\frac{1}{6}$ of the floor area for waste.

11. How many square feet are covered by a board 22 ft. long and 10 in. wide?

12. If a board 18 ft. long and 9 in. wide and 3 in. thick is cut into boards 1 in. in thickness, how many square feet do they cover?

13. How many board feet in 570 cu. ft.?

14. How many feet of wood will be required for a fence 1 mi. long, 5 boards high, each board being 4 in. wide and 1 in. thick?

15. A banquet hall 60 ft. long and 35 ft. 8 in. wide is to be covered with maple flooring costing \$36 per 1000 ft. A man getting \$5 per day can lay 200 sq. ft. Find the cost of flooring the hall.

16. An apartment of 5 rooms is to be covered with yellow pine flooring costing \$20 per 1000 ft. The dimensions of the rooms are:

8 ft. 10 in. \times 7 ft. 6 in.

7 ft. 10 in. \times 7 ft.

12 ft. \times 10 ft. 9 in.

6 ft. 6 in. \times 5 ft. 6 in.

6 ft. 10 in. \times 5 ft. 4 in.

Find the total cost of the lumber.

17. How many board feet of lumber are in a pile 8 ft. high, 9 ft. wide, and 20 ft. long? Find the value of the lumber at \$15 per 1000 ft.

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19. How many board feet in 32 joists, each joist being 20 ft. long and 5 in. square?

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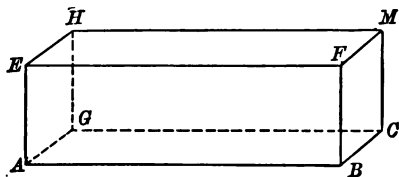


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The lines AB , BC , AE , etc., represent the **edges** of a solid.

The surfaces $AEFB$, $BCMF$, $CMHG$, and $GHEA$ are the **lateral faces** of the solid.

6. Hexagonal prism : area of base, $12\frac{1}{2}$ sq. ft.; altitude, $2\frac{1}{2}$ ft.

Find the volume of the following cylinders :

7. Area of base, 16 sq. ft. ; altitude 5 ft.

8. Area of base, 25 sq. m. ; altitude 4 m.

PROBLEMS

346. 1. The dimensions of the base of a rectangular prism are 5 ft. by 3 ft. Find the volume of the prism, if the altitude is 4 ft.

2. The area of the base of a cylinder is 30 sq. ft. The cylinder is 6 ft. high. Find the volume.

3. A column of stone in the form of a rectangular prism is 8 ft. high. The area of the base is 40 sq. ft. Find the weight of the column, if 1 cu. ft. weighs 160 lb.

4. One square prism has a base 4 ft. long. The prism is 7 ft. high. Find the volume of the prism.

5. Find the volume of a square prism that has the same base as the one given in No. 4, but is twice as high.

6. What is the ratio of the volume of the prism in No. 4 to the volume of the prism in No. 5 ?

7. Find the volume of a square prism that has the same height as the prism given in No. 4, but has a base 8 ft. long.

8. What is the ratio of the volume of the prism in No. 4 to the volume of the prism in No. 7 ?

9. Find the volume of a cylinder that is 4 ft. high and has a base 16 in. in diameter.

10. Find the volume of a cylinder that has the same height as the one given in No. 9, but has a diameter twice as great.

11. Find the volume of a cylinder that has the same base as the one given in No. 9, but is twice as high.

12. Compare the answers in 9 and 10; in 9 and 11.

13. How many cubic inches of copper in a copper wire $\frac{1}{16}$ in. in diameter and 360 ft. long?

14. How many cubic feet of iron in an iron wire 2000 ft. long and $\frac{1}{4}$ in. in diameter?

PRACTICAL APPLICATIONS

To find the Number of Gallons in a Cistern

347. What is the capacity in gallons of a cistern 8 ft. \times 3 ft. \times 11 ft.?

Process

231 cu. in. = 1 gal.

$$\frac{1}{231} \text{ of } (8 \times 3 \times 11) \times 1728$$

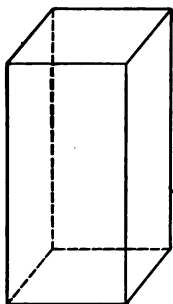
$$\frac{21}{7} = \frac{13824}{7} = 1974\frac{6}{7} \text{ gal.}$$

Explanation. Find the capacity (volume or solid contents) of the cistern. Since one liquid gallon contains 231 cu. in., the number of gallons the cistern holds is $\frac{1}{231}$ of the number of cubic inches in the cistern.

The contents are $8 \times 3 \times 11 = 264$ cu. ft. Change to cubic inches. 264×1728 cu. in. = 456,192 cu. in. $\frac{1}{231}$ of 456,192 = $1974\frac{6}{7}$ gal.

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SURFACES OF SOLIDS

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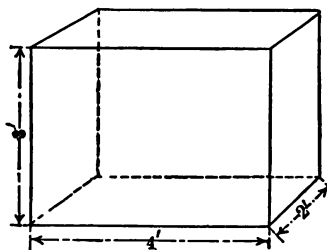
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Lateral surface = 36 sq. ft. *Ans.*



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To find the Area of the Convex Surface of a
Circular Cylinder



FIG. 1

1. A cylinder is 6 ft. high; the diameter of the base is 2 ft. Find the convex surface.

Wrap a piece of paper around the cylinder so that it will equal the convex surface; then lay the paper as shown in Fig. 2. The paper will have the shape of a rectangle. The area of the rectangle, which is equal to the convex surface of the cylinder, is found by multiplying the length (the circumference of the base) by the altitude (the altitude of the cylinder).

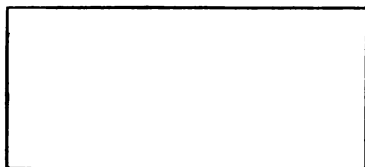


FIG. 2

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337. RULE. The area of the convex surface of a circular cylinder equals the circumference of the base multiplied by the altitude, expressed in like units.

Formula $CS = 2 \pi \times r \times h.$

EXERCISES

338. Find the area of the lateral surface of the following rectangular prisms :

Base	Altitude
1. 3 ft. by 6 ft.	5 ft.
2. 2 ft. by 3 ft.	4 ft.
3. $2\frac{1}{2}$ m. by $3\frac{1}{2}$ m.	$2\frac{1}{2}$ m.
4. 9 dm. by 10 dm.	2 dm.

Find the area of the lateral surface of the following prisms :

5. Triangular prism ; perimeter of base, 18 ft. ; altitude, 3 ft.

6. Hexagonal prism ; perimeter of base, 12 ft. ; altitude, 3 ft.

Find the area of the lateral surface of the following circular cylinders :

7. Circumference of base, 21 in. ; altitude, 6 in.

8. Diameter of base, 2 ft. ; altitude, 1 ft.

9. Find the total surface of the circular cylinders in exercises 7 and 8.

PROBLEMS

339. 1. How much sheet iron is necessary to line the sides of a refrigerator 18 ft. long, 12 ft. wide, and 8 ft. high, adding 10 % for waste ?

2. Find the total surface of a cube 15 in. long.

3. The distance around a cylindrical gas tank is 154 ft. What is the lateral surface of the tank, if the altitude is 40 ft.?

4. How much sheet iron will be needed to make a smokestack 20 ft. high and 7 ft. in diameter?

5. Find the lateral surface of a rectangular prism whose base is a square 3 ft. long and whose height is 6 ft.

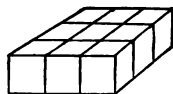
VOLUMES OF SOLIDS

To find the Volume of a Prism



340. 1. By using inch cubes, build a rectangular prism 3 in. long, 3 in. wide, and 3 in. high.

2. This prism can be divided horizontally into 3 layers, or prisms, each being 3 in. long, 3 in. wide, and 1 in. high. The volume of 1 layer is 9 cu. in.



Therefore, the volume of the whole prism is 3×9 cu. in.
= 27 cu. in.

341. RULE. The volume of a prism equals the product of the area of the base by the altitude; the result, expressed as cubic units of the same denomination as the dimensions, is the volume.

NOTE. The volume of a rectangular prism may be found by multiplying together the length, width, and altitude, the three dimensions being expressed in like units.

EQUATION AND FORMULA.

$$\text{Volume Prism} = l \times w \times a.$$

To find the Volume of a Cube

342. In a cube the length, width, and altitude are equal.

RULE. The volume of a cube equals the product obtained by taking the length 3 times as a factor.

EQUATION AND FORMULA.

$$\text{Volume Cube} = l^3 \text{ or } (l \times l \times l).$$

To find the Volume of a Cylinder

343. RULE. The volume of a cylinder equals the product of the area of the base by the altitude.

The two rules for finding the volumes of a prism and a cylinder may be combined into one rule.

344. RULE. The volume of a prism or a cylinder equals the product of the area of the base by the altitude.

EXERCISES

345. Find the volume of the following rectangular prisms:

Length	Width	Altitude	Length	Width	Altitude
1. 3 ft.	2 ft.	1 ft.	2. 3 ft.	3 ft.	4 ft.
3. 9 in.	2 ft.	6 in.	4. 3 m.	2 m.	1 m.

Find the volume of the following prisms:

5. Triangular prism: area of base, 30 sq. ft.; altitude, 4 ft.

3. The distance around a cylindrical gas tank is 154 ft. What is the lateral surface of the tank, if the altitude is 40 ft.?

4. How much sheet iron will be needed to make a smokestack 20 ft. high and 7 ft. in diameter?

5. Find the lateral surface of a rectangular prism whose base is a square 3 ft. long and whose height is 6 ft.

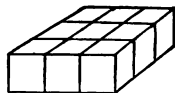
VOLUMES OF SOLIDS

To find the Volume of a Prism



340. 1. By using inch cubes, build a rectangular prism 3 in. long, 3 in. wide, and 3 in. high.

2. This prism can be divided horizontally into 3 layers, or prisms, each being 3 in. long, 3 in. wide, and 1 in. high. The volume of 1 layer is 9 cu. in.



Therefore, the volume of the whole prism is 3×9 cu. in.
= 27 cu. in.

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NOTE. The volume of a rectangular prism may be found by multiplying together the length, width, and altitude, the three dimensions being expressed in like units.

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$$\text{Volume Prism} = l \times w \times a.$$

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342. In a cube the length, width, and altitude are equal.

RULE. The volume of a cube equals the product obtained by taking the length 3 times as a factor.

EQUATION AND FORMULA.

$$\text{Volume Cube} = l^3 \text{ or } (l \times l \times l).$$

To find the Volume of a Cylinder

343. RULE. The volume of a cylinder equals the product of the area of the base by the altitude.

The two rules for finding the volumes of a prism and a cylinder may be combined into one rule.

344. RULE. The volume of a prism or a cylinder equals the product of the area of the base by the altitude.

EXERCISES

345. Find the volume of the following rectangular prisms:

Length	Width	Altitude	Length	Width	Altitude
1. 3 ft.	2 ft.	1 ft.	2. 3 ft.	3 ft.	4 ft.
3. 9 in.	2 ft.	6 in.	4. 3 m.	2 m.	1 m.

Find the volume of the following prisms:

5. Triangular prism: area of base, 30 sq. ft. · altitude, 4 ft.

6. Hexagonal prism : area of base, $12\frac{1}{2}$ sq. ft. ; altitude, $2\frac{1}{2}$ ft.

Find the volume of the following cylinders :

7. Area of base, 16 sq. ft. ; altitude 5 ft.

8. Area of base, 25 sq. m. ; altitude 4 m.

PROBLEMS

346. 1. The dimensions of the base of a rectangular prism are 5 ft. by 3 ft. Find the volume of the prism, if the altitude is 4 ft.

2. The area of the base of a cylinder is 30 sq. ft. The cylinder is 6 ft. high. Find the volume.

3. A column of stone in the form of a rectangular prism is 8 ft. high. The area of the base is 40 sq. ft. Find the weight of the column, if 1 cu. ft. weighs 160 lb.

4. One square prism has a base 4 ft. long. The prism is 7 ft. high. Find the volume of the prism.

5. Find the volume of a square prism that has the same base as the one given in No. 4, but is twice as high.

6. What is the ratio of the volume of the prism in No. 4 to the volume of the prism in No. 5 ?

7. Find the volume of a square prism that has the same height as the prism given in No. 4, but has a base 8 ft. long.

8. What is the ratio of the volume of the prism in No. 4 to the volume of the prism in No. 7 ?

9. Find the volume of a cylinder that is 4 ft. high and has a base 16 in. in diameter.

10. Find the volume of a cylinder that has the same height as the one given in No. 9, but has a diameter twice as great.

11. Find the volume of a cylinder that has the same base as the one given in No. 9, but is twice as high.

12. Compare the answers in 9 and 10; in 9 and 11.

13. How many cubic inches of copper in a copper wire $\frac{1}{16}$ in. in diameter and 360 ft. long?

14. How many cubic feet of iron in an iron wire 2000 ft. long and $\frac{1}{4}$ in. in diameter?

PRACTICAL APPLICATIONS

To find the Number of Gallons in a Cistern

347. What is the capacity in gallons of a cistern 8 ft. \times 3 ft. \times 11 ft.?

Process

231 cu. in. = 1 gal.

$$\begin{array}{r} \frac{1}{231} \text{ of } (8 \times 3 \times 11) \times 1728 \\ \frac{21}{7} = \frac{13824}{7} = 1974\frac{6}{7} \text{ gal.} \end{array}$$

Explanation. Find the capacity (volume or solid contents) of the cistern. Since one liquid gallon contains 231 cu. in., the number of gallons the cistern holds is $\frac{1}{231}$ of the number of cubic inches in the cistern.

The contents are $8 \times 3 \times 11 = 264$ cu. ft. Change to cubic inches. 264×1728 cu. in. = 456,192 cu. in. $\frac{1}{231}$ of 456,192 = $1974\frac{6}{7}$ gal.

Short Process

$$(8 \times 3 \times 11) \times 7\frac{1}{2} = 1980 \text{ gal.}$$

Explanation. The number of gallons is approximately $7\frac{1}{2}$ times the number of cubic feet.

To find the Number of Bushels in a Bin

348. Find the number of bushels in a bin 5 ft. long, 4 ft. wide, and 3 ft. high or deep.	Process $2150 \text{ cu. in.}^* = 1 \text{ bu.}$ $\frac{1}{1250}$ of $5 \times 4 \times 3 \times 1728$ $\frac{430}{215} = 48\frac{48}{215} \text{ bu.} \quad \text{Ans.}$
---	---

Explanation. One bushel contains 2150.42 cu. in. Therefore, the number of bushels is approximately $\frac{1}{2150}$ of the number of cubic inches.

Short Process

$$\frac{5}{4} \text{ cu. ft.} = 1 \text{ bu.}$$

$$4 \times \frac{5}{4} \times 4 \times 3 = 48 \text{ bu.} \quad \text{Ans.}$$

Explanation. 2150 cu. in. is approximately $\frac{5}{4}$ cu. ft. Each bushel occupies about $\frac{5}{4}$ cu. ft. Therefore, the number of bushels is $\frac{4}{5}$ of the number of cubic feet.

WRITTEN EXERCISES

349. Find the number of gallons that can be stored in the following cisterns:

1. 15 ft. \times 5 ft. \times 2 ft. 6 in.
2. 6 ft. \times 4 ft. \times 4 ft.
3. 3 ft. \times 5 ft. \times 3 ft.
4. 2 ft. 6 in. \times 3 ft. 6 in. \times 2 ft.

* NOTE. In a bushel there are 2150.42 cu. in. 2150 is an approximation.

Find the number of bushels that can be stored in the following bins:

5. 2 ft. \times 3 ft. \times 9 in.

6. 10 ft. \times 3 ft. \times 12 in.

7. 8 ft. \times $7\frac{1}{2}$ ft. \times 8 ft.

8. 4 ft. \times 3 ft. 3 in. \times 2 ft.

9. How many gallons will a vat hold that is 4 ft. square and 12 in. high?

10. Find the cost of digging a cellar 25 ft. by 10 ft., and 4 ft. deep, at 75 ct. a cubic yard.

11. Find the cost of carting away the excavated earth at 25 ct. a cartload, each cart holding $1\frac{1}{4}$ cu. yd.

12. The weight of ice is .92 of the weight of water. What is the weight of a cake of ice 3 ft. long, 2 ft. wide, and 1 ft. high? (1 cu. ft. of water weighs $62\frac{1}{2}$ lb.)

13. One tank is 6 ft. high; the radius of its base is 4 ft. How many gallons does it hold?

14. Find the capacity of a tank that is as high as the one given in No. 13, but has a base whose radius is twice as great.

15. A box car is 33 ft. long, 8 ft. 8 in. wide, 7 ft. 6 in. high, inside measure.

(a) How many cubic feet in the car?

(b) How many bushels will the car hold?

350. Find the value for ? in each of the following :

	FIGURE	BASE	ALTITUDE	RADIUS	CIRCUM-FERENCE	AREA
1.	Rectangle	6 ft.	6 in.	—	—	?
2.	Rectangle	?	2 ft.	—	—	18 sq. ft.
3.	Square	?	?	—	—	256 sq. ft.
4.	Parallelogram	1½ ft.	9 in.	—	—	?
5.	Parallelogram	?	12 ft.	—	—	216 sq. ft.
6.	Parallelogram	24 ft.	?	—	—	384 sq. ft.
7.	Rectangle	?	5½ ft.	—	—	41½ sq. ft.
8.	Rectangle	10½ ft.	?	—	—	26½ sq. ft.
9.	Circle	—	—	7 ft.	?	?
10.	Circle	—	—	?	?	44 sq. in.
11.	Circle	—	—	?	22 in.	?
12.	Circle	—	—	9½ in.	?	?
13.	Triangle	9½ ft.	2 yd.	—	—	?
14.	Triangle	?	3 yd.	—	—	180 sq. ft.
15.	Triangle	2.8 ft.	?	—	—	9.8 sq. ft.
16.	Triangle	?	7 in.	—	—	9 sq. ft.
17.	Rectangle	12.6 ft.	2.7 yd.	—	—	?
18.	Triangle	5.6 ft.	1.3 yd.	—	—	?
19.	Square	?	?	—	—	42.25 sq. yd.
20.	Parallelogram	?	8.5 ft.	—	—	21.25 sq. ft.
21.	Circle	—	—	2.5 ft.	?	?
22.	Circle	—	—	?	?	36 sq. ft.
23.	Triangle	40.5 ft.	?	—	—	101.25 sq. ft.
24.	Square	?	?	—	—	21.16 sq. ft.
25.	Rectangle	?	8.9 ft.	—	—	30.26 sq. ft.

351. Find the value for ? in each of the following :

	FIGURE	PERIMETER OF BASE	ALTITUDE	SLANT HEIGHT	DIAMETER	CONVEX SURFACE
1.	Prism	16 sq. ft.	2 yd.	—	—	?
2.	Prism	24 sq. ft.	?	—	—	120 sq. ft.
3.	Cylinder	18.5 sq. ft.	1½ yd.	—	—	?

352. Find the value for ? in each of the following :

	FIGURE	AREA OF BASE	ALTITUDE	RADIUS	VOLUME
1.	Prism	48 sq. ft.	$2\frac{1}{2}$ yd.	—	?
2.	Cylinder	72 sq. in.	$1\frac{1}{2}$ ft.	—	?

MISCELLANEOUS PROBLEMS IN MENSURATION

353. 1. How many lots, each 25 ft. by 100 ft., can be made from a plot 300 ft. by 200 ft. ? (Draw a diagram.)

2. The area of a rectangular plot is 1200 sq. ft. The base is three times as long as the altitude. Find the dimensions of the plot.

3. A rectangular field is 180 yd. long; the width is 40 % of the length. Find the area.

4. A triangular flag is made of two strips of felt of equal size. The base of the flag is 9 in.; the altitude of the flag is 18 in. How many square inches of felt in each strip?

5. A circular track is 140 ft. in diameter. How many times must a boy run around the track to travel a mile ? ($R = 3\frac{1}{4}$.)

6. A bin 10 ft. long, 5 ft. wide, and 5 ft. deep is filled with wheat.

(a) How many bushels of wheat are in the bin ?

(b) Find the value of the wheat at \$1.05 per bushel.

7. A rectangular stone prism is to have all its faces polished except the base. The base of the prism is 3 ft. 4 in. long and 2 ft. 10 in. wide. What will the work cost at 75 ct. a sq. ft.?

8. A cylindrical stone column is 12 ft. high; the area of the base is 12 sq. ft.

(a) Find the volume of the column in cubic yards.

(b) Find the weight of the column if one cubic yard weighs 42 lb.

(c) Find the area of the convex surface of the column.

(d) Find the area of the total surface of the column.

336. RULE. The area of the convex surface of a prism equals the perimeter of the base multiplied by the altitude, expressed in like units.

To find the Area of the Convex Surface of a
Circular Cylinder



FIG. 1

1. A cylinder is 6 ft. high; the diameter of the base is 2 ft. Find the convex surface.

Wrap a piece of paper around the cylinder so that it will equal the convex surface; then lay the paper as shown in Fig. 2. The paper will have the shape of a rectangle. The area of the rectangle, which is equal to the convex surface of the cylinder, is found by multiplying the length (the circumference of the base) by the altitude (the altitude of the cylinder).

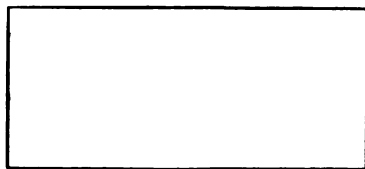


FIG. 2

Convex Surface of Circular Cylinder = $3.1416 \times 2 \times 6 = 37.6992$ (sq. ft.).

337. RULE. The area of the convex surface of a circular cylinder equals the circumference of the base multiplied by the altitude, expressed in like units.

Formula $CS = 2\pi \times r \times h$.

EXERCISES

338. Find the area of the lateral surface of the following rectangular prisms :

Base	Altitude
1. 3 ft. by 6 ft.	5 ft.
2. 2 ft. by 3 ft.	4 ft.
3. $2\frac{1}{2}$ m. by $3\frac{1}{2}$ m.	$2\frac{1}{2}$ m.
4. 9 dm. by 10 dm.	2 dm.

Find the area of the lateral surface of the following prisms :

5. Triangular prism ; perimeter of base, 18 ft. ; altitude, 3 ft.

6. Hexagonal prism ; perimeter of base, 12 ft. ; altitude, 3 ft.

Find the area of the lateral surface of the following circular cylinders :

7. Circumference of base, 21 in. ; altitude, 6 in.

8. Diameter of base, 2 ft. ; altitude, 1 ft.

9. Find the total surface of the circular cylinders in exercises 7 and 8.

PROBLEMS

339. 1. How much sheet iron is necessary to line the sides of a refrigerator 18 ft. long, 12 ft. wide, and 8 ft. high, adding 10 % for waste ?

2. Find the total surface of a cube 15 in. long.

3. The distance around a cylindrical gas tank is 154 ft. What is the lateral surface of the tank, if the altitude is 40 ft.?

4. How much sheet iron will be needed to make a smokestack 20 ft. high and 7 ft. in diameter?

5. Find the lateral surface of a rectangular prism whose base is a square 3 ft. long and whose height is 6 ft.

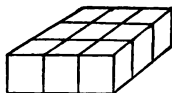
VOLUMES OF SOLIDS

To find the Volume of a Prism



340. 1. By using inch cubes, build a rectangular prism 3 in. long, 3 in. wide, and 3 in. high.

2. This prism can be divided horizontally into 3 layers, or prisms, each being 3 in. long, 3 in. wide, and 1 in. high. The volume of 1 layer is 9 cu. in.



Therefore, the volume of the whole prism is 3×9 cu. in.
= 27 cu. in.

341. RULE. The volume of a prism equals the product of the area of the base by the altitude; the result, expressed as cubic units of the same denomination as the dimensions, is the volume.

NOTE. The volume of a rectangular prism may be found by multiplying together the length, width, and altitude, the three dimensions being expressed in like units.

EQUATION AND FORMULA.

$$\text{Volume Prism} = l \times w \times a.$$

To find the Volume of a Cube

342. In a cube the length, width, and altitude are equal.

RULE. The volume of a cube equals the product obtained by taking the length 3 times as a factor.

EQUATION AND FORMULA.

$$\text{Volume Cube} = l^3 \text{ or } (l \times l \times l).$$

To find the Volume of a Cylinder

343. RULE. The volume of a cylinder equals the product of the area of the base by the altitude.

The two rules for finding the volumes of a prism and a cylinder may be combined into one rule.

344. RULE. The volume of a prism or a cylinder equals the product of the area of the base by the altitude.

EXERCISES

345. Find the volume of the following rectangular prisms:

Length	Width	Altitude	Length	Width	Altitude
1. 3 ft.	2 ft.	1 ft.	2. 3 ft.	3 ft.	4 ft.
3. 9 in.	2 ft.	6 in.	4. 3 m.	2 m.	1 m.

Find the volume of the following prisms:

5. Triangular prism: area of base, 30 sq. ft.; altitude, 4 ft.

6. Hexagonal prism : area of base, $12\frac{1}{2}$ sq. ft.; altitude, $2\frac{1}{2}$ ft.

Find the volume of the following cylinders :

7. Area of base, 16 sq. ft.; altitude 5 ft.

8. Area of base, 25 sq. m.; altitude 4 m.

PROBLEMS

346. 1. The dimensions of the base of a rectangular prism are 5 ft. by 3 ft. Find the volume of the prism, if the altitude is 4 ft.

2. The area of the base of a cylinder is 30 sq. ft. The cylinder is 6 ft. high. Find the volume.

3. A column of stone in the form of a rectangular prism is 8 ft. high. The area of the base is 40 sq. ft. Find the weight of the column, if 1 cu. ft. weighs 160 lb.

4. One square prism has a base 4 ft. long. The prism is 7 ft. high. Find the volume of the prism.

5. Find the volume of a square prism that has the same base as the one given in No. 4, but is twice as high.

6. What is the ratio of the volume of the prism in No. 4 to the volume of the prism in No. 5?

7. Find the volume of a square prism that has the same height as the prism given in No. 4, but has a base 8 ft. long.

8. What is the ratio of the volume of the prism in No. 4 to the volume of the prism in No. 7?

9. Find the volume of a cylinder that is 4 ft. high and has a base 16 in. in diameter.

10. Find the volume of a cylinder that has the same height as the one given in No. 9, but has a diameter twice as great.

11. Find the volume of a cylinder that has the same base as the one given in No. 9, but is twice as high.

12. Compare the answers in 9 and 10; in 9 and 11.

13. How many cubic inches of copper in a copper wire $\frac{1}{16}$ in. in diameter and 360 ft. long?

14. How many cubic feet of iron in an iron wire 2000 ft. long and $\frac{1}{4}$ in. in diameter?

PRACTICAL APPLICATIONS

To find the Number of Gallons in a Cistern

347. What is the capacity in gallons of a cistern 8 ft. \times 3 ft. \times 11 ft.?

Process

$$231 \text{ cu. in.} = 1 \text{ gal.}$$

$$\frac{1}{231} \text{ of } (8 \times 3 \times 11) \times 1728$$

$$\frac{21}{7} = \frac{13824}{7} = 1974\frac{6}{7} \text{ gal.}$$

Explanation. Find the capacity (volume or solid contents) of the cistern. Since one liquid gallon contains 231 cu. in., the number of gallons the cistern holds is $\frac{1}{231}$ of the number of cubic inches in the cistern.

The contents are $8 \times 3 \times 11 = 264$ cu. ft. Change to cubic inches. 264×1728 cu. in. $= 456,192$ cu. in. $\frac{1}{231}$ of $456,192 = 1974\frac{6}{7}$ gal.

10. A hall is 100 ft. long and 8 ft. wide. Find the cost of laying a floor at \$2.90 per thousand square feet, allowing for a loss of $\frac{1}{6}$ of the floor area for waste.

11. How many square feet are covered by a board 22 ft. long and 10 in. wide?

12. If a board 18 ft. long and 9 in. wide and 3 in. thick is cut into boards 1 in. in thickness, how many square feet do they cover?

13. How many board feet in 570 cu. ft.?

14. How many feet of wood will be required for a fence 1 mi. long, 5 boards high, each board being 4 in. wide and 1 in. thick?

15. A banquet hall 60 ft. long and 35 ft. 8 in. wide is to be covered with maple flooring costing \$36 per 1000 ft. A man getting \$5 per day can lay 200 sq. ft. Find the cost of flooring the hall.

16. An apartment of 5 rooms is to be covered with yellow pine flooring costing \$20 per 1000 ft. The dimensions of the rooms are:

8 ft. 10 in. \times 7 ft. 6 in.

7 ft. 10 in. \times 7 ft.

12 ft. \times 10 ft. 9 in.

6 ft. 6 in. \times 5 ft. 6 in.

6 ft. 10 in. \times 5 ft. 4 in.

Find the total cost of the lumber.

17. How many board feet of lumber are in a pile 8 ft. high, 9 ft. wide, and 20 ft. long? Find the value of the lumber at \$15 per 1000 ft.

18. How many feet, board measure, in 22 boards, each board being 19 ft. long, 10 in. wide, and 1 in. thick?

19. How many board feet in 32 joists, each joist being 20 ft. long and 5 in. square?

20. How many feet of lumber will be required for a box 8 ft. high, 4 ft. square?

SOLIDS

FOR READING AND DISCUSSION

328. Anything that occupies space is a solid; *e.g.* a baseball, a book, a pencil, etc. Solids have three dimensions, namely, length, breadth, and thickness;

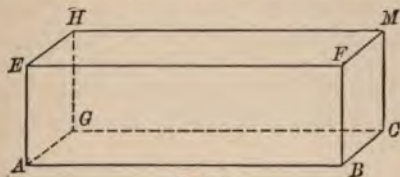


FIG. 1.

surfaces have only two dimensions, length and breadth; lines have only one dimension, length.

Fig. 1 is not a solid, but it represents a solid.

The lines AB , BC , AE , etc., represent the edges of a solid.

The surfaces $A E F B$, $B C M F$, $C M H G$, and $G H E A$ are the lateral faces of the solid.

329. A **rectangular solid** is a solid having 6 rectangular faces or surfaces. Fig. 1 represents a rectangular solid. The lower and upper faces ($ABCG$ and $EFMH$) are called the lower and upper **bases** of the solid.

330. A **prism** is a solid whose lateral faces are parallelograms, and whose upper and lower bases are equal and parallel to each other.



Rectangular Prism



Triangular Prism

Prisms are named from the shape of their bases — as triangular prisms, rectangular prisms, square prisms, hexagonal prisms, etc.

331. By the **lateral surface** of a prism is meant the total surface of all the faces except the upper and lower bases.



332. A **circular cylinder** is a solid bounded by a uniformly curved surface and having circles parallel to each other for its upper and lower bases.

333. The **lateral surface** of a cylinder is the curved surface.

334. The total surface of a prism or a cylinder consists of the lateral surface plus the upper and lower bases.

SURFACES OF SOLIDS

To find the Area of the Lateral Surface of a Prism

335. A rectangular solid is 4 ft. long, 2 ft. wide, and 3 ft. high. Find the lateral surface.

Process

The lateral surface consists of the areas of 4 rectangles.

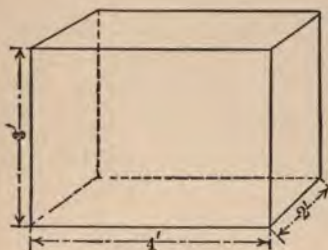
Area of one rectangle, 4 ft. by 3 ft. = 12 sq. ft.

Area of one rectangle, 4 ft. by 3 ft. = 12 sq. ft.

Area of one rectangle, 2 ft. by 3 ft. = 6 sq. ft.

Area of one rectangle, 2 ft. by 3 ft. = 6 sq. ft.

Lateral surface = 36 sq. ft. *Ans.*



A shorter method is to find the perimeter of the base (12 ft.) and multiply it by the height. The result, expressed in square units of the same denominations as the dimensions, is the lateral surface.

336. RULE. The area of the convex surface of a prism equals the perimeter of the base multiplied by the altitude, expressed in like units.

To find the Area of the Convex Surface of a
Circular Cylinder



FIG. 1

1. A cylinder is 6 ft. high; the diameter of the base is 2 ft. Find the convex surface.

Wrap a piece of paper around the cylinder so that it will equal the convex surface; then lay the paper as shown in Fig. 2. The paper will have the shape of a rectangle. The area of the rectangle, which is equal to the convex surface of the cylinder, is found by multiplying the length (the circumference of the base) by the altitude (the altitude of the cylinder).

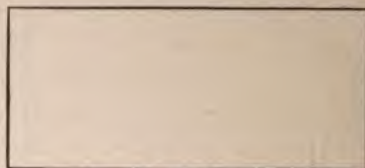


FIG. 2

Convex Surface of Circular Cylinder = $3.1416 \times 2 \times 6 = 37.6992$ (sq. ft.).

337. RULE. The area of the convex surface of a circular cylinder equals the circumference of the base multiplied by the altitude, expressed in like units.

Formula $CS = 2\pi \times r \times h$.

EXERCISES

338. Find the area of the lateral surface of the following rectangular prisms :

Base	Altitude
1. 3 ft. by 6 ft.	5 ft.
2. 2 ft. by 3 ft.	4 ft.
3. $2\frac{1}{2}$ m. by $3\frac{1}{2}$ m.	$2\frac{1}{2}$ m.
4. 9 dm. by 10 dm.	2 dm.

Find the area of the lateral surface of the following prisms :

5. Triangular prism ; perimeter of base, 18 ft. ; altitude, 3 ft.

6. Hexagonal prism ; perimeter of base, 12 ft. ; altitude, 3 ft.

Find the area of the lateral surface of the following circular cylinders :

7. Circumference of base, 21 in. ; altitude, 6 in.

8. Diameter of base, 2 ft. ; altitude, 1 ft.

9. Find the total surface of the circular cylinders in exercises 7 and 8.

PROBLEMS

339. 1. How much sheet iron is necessary to line the sides of a refrigerator 18 ft. long, 12 ft. wide, and 8 ft. high, adding 10 % for waste ?

2. Find the total surface of a cube 15 in. long.

3. The distance around a cylindrical gas tank is 154 ft. What is the lateral surface of the tank, if the altitude is 40 ft.?

4. How much sheet iron will be needed to make a smokestack 20 ft. high and 7 ft. in diameter?

5. Find the lateral surface of a rectangular prism whose base is a square 3 ft. long and whose height is 6 ft.

VOLUMES OF SOLIDS

To find the Volume of a Prism



340. 1. By using inch cubes, build a rectangular prism 3 in. long, 3 in. wide, and 3 in. high.

2. This prism can be divided horizontally into 3 layers, or prisms, each being 3 in. long, 3 in. wide, and 1 in. high. The volume of 1 layer is 9 cu. in.



Therefore, the volume of the whole prism is 3×9 cu. in.
 $= 27$ cu. in.

341. RULE. The volume of a prism equals the product of the area of the base by the altitude; the result, expressed as cubic units of the same denomination as the dimensions, is the volume.

NOTE. The volume of a rectangular prism may be found by multiplying together the length, width, and altitude, the three dimensions being expressed in like units.

EQUATION AND FORMULA.

$$\text{Volume Prism} = l \times w \times a.$$

To find the Volume of a Cube

342. In a cube the length, width, and altitude are equal.

RULE. The volume of a cube equals the product obtained by taking the length 3 times as a factor.

EQUATION AND FORMULA.

$$\text{Volume Cube} = l^3 \text{ or } (l \times l \times l).$$

To find the Volume of a Cylinder

343. RULE. The volume of a cylinder equals the product of the area of the base by the altitude.

The two rules for finding the volumes of a prism and a cylinder may be combined into one rule.

344. RULE. The volume of a prism or a cylinder equals the product of the area of the base by the altitude.

EXERCISES

345. Find the volume of the following rectangular prisms:

	Length	Width	Altitude		Length	Width	Altitude
1.	3 ft.	2 ft.	1 ft.	2.	3 ft.	3 ft.	4 ft.
3.	9 in.	2 ft.	6 in.	4.	3 m.	2 m.	1 m.

Find the volume of the following prisms:

5. Triangular prism: area of base, 30 sq. ft.: altitude, 4 ft.

6. Hexagonal prism: area of base, $12\frac{1}{2}$ sq. ft.; altitude, $2\frac{1}{2}$ ft.

Find the volume of the following cylinders:

7. Area of base, 16 sq. ft.; altitude 5 ft.

8. Area of base, 25 sq. m.; altitude 4 m.

PROBLEMS

346. 1. The dimensions of the base of a rectangular prism are 5 ft. by 3 ft. Find the volume of the prism, if the altitude is 4 ft.

2. The area of the base of a cylinder is 30 sq. ft. The cylinder is 6 ft. high. Find the volume.

3. A column of stone in the form of a rectangular prism is 8 ft. high. The area of the base is 40 sq. ft. Find the weight of the column, if 1 cu. ft. weighs 160 lb.

4. One square prism has a base 4 ft. long. The prism is 7 ft. high. Find the volume of the prism.

5. Find the volume of a square prism that has the same base as the one given in No. 4, but is twice as high.

6. What is the ratio of the volume of the prism in No. 4 to the volume of the prism in No. 5?

7. Find the volume of a square prism that has the same height as the prism given in No. 4, but has a base 8 ft. long.

8. What is the ratio of the volume of the prism in No. 4 to the volume of the prism in No. 7?

9. Find the volume of a cylinder that is 4 ft. high and has a base 16 in. in diameter.

10. Find the volume of a cylinder that has the same height as the one given in No. 9, but has a diameter twice as great.

11. Find the volume of a cylinder that has the same base as the one given in No. 9, but is twice as high.

12. Compare the answers in 9 and 10; in 9 and 11.

13. How many cubic inches of copper in a copper wire $\frac{1}{16}$ in. in diameter and 360 ft. long?

14. How many cubic feet of iron in an iron wire 2000 ft. long and $\frac{1}{4}$ in. in diameter?

PRACTICAL APPLICATIONS

To find the Number of Gallons in a Cistern

347. What is the capacity in gallons of a cistern 8 ft. \times 3 ft. \times 11 ft.?

Process

231 cu. in. = 1 gal.

$$\frac{1}{231} \text{ of } (8 \times 3 \times 11) \times 1728$$

$$\frac{21}{7} = \frac{13824}{7} = 1974\frac{6}{7} \text{ gal.}$$

Explanation. Find the capacity (volume or solid contents) of the cistern. Since one liquid gallon contains 231 cu. in., the number of gallons the cistern holds is $\frac{1}{231}$ of the number of cubic inches in the cistern.

The contents are $8 \times 3 \times 11 = 264$ cu. ft. Change to cubic inches. 264×1728 cu. in. = 456,192 cu. in. $\frac{1}{231}$ of 456,192 = $1974\frac{6}{7}$ gal.

Short Process

$$(8 \times 3 \times 11) \times 7\frac{1}{2} = 1980 \text{ gal.}$$

Explanation. The number of gallons is approximately $7\frac{1}{2}$ times the number of **cubic feet**.

To find the Number of Bushels in a Bin

348. Find the number of bushels in a bin 5 ft. long, 4 ft. wide, and 3 ft. high or deep.	<div style="text-align: center;">Process</div> $2150 \text{ cu. in.}^* = 1 \text{ bu.}$ $\frac{1}{1250} \text{ of } 5 \times 4 \times 3 \times 1728$ $\frac{430}{215} = 48\frac{48}{215} \text{ bu. } \textit{Ans.}$
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Explanation. One bushel contains 2150.42 cu. in. Therefore, the number of bushels is approximately $\frac{1}{2150}$ of the number of **cubic inches**.

Short Process

$$\frac{5}{4} \text{ cu. ft.} = 1 \text{ bu.}$$

$$4 \times 5 \times 4 \times 3 = 48 \text{ bu. } \textit{Ans.}$$

$$\frac{5}{4}$$

Explanation. 2150 cu. in. is approximately $\frac{5}{4}$ cu. ft. Each bushel occupies about $\frac{5}{4}$ cu. ft. Therefore, the number of bushels is $\frac{4}{5}$ of the number of **cubic feet**.

WRITTEN EXERCISES

349. Find the number of gallons that can be stored in the following cisterns:

1. 15 ft. \times 5 ft. \times 2 ft. 6 in.
2. 6 ft. \times 4 ft. \times 4 ft.
3. 3 ft. \times 5 ft. \times 3 ft.
4. 2 ft. 6 in. \times 3 ft. 6 in. \times 2 ft.

* NOTE. In a bushel there are 2150.42 cu. in. 2150 is an approximation.

Find the number of bushels that can be stored in the following bins :

5. $2 \text{ ft.} \times 3 \text{ ft.} \times 9 \text{ in.}$

6. $10 \text{ ft.} \times 3 \text{ ft.} \times 12 \text{ in.}$

7. $8 \text{ ft.} \times 7\frac{1}{2} \text{ ft.} \times 8 \text{ ft.}$

8. $4 \text{ ft.} \times 3 \text{ ft.} \times 3 \text{ in.} \times 2 \text{ ft.}$

9. How many gallons will a vat hold that is 4 ft. square and 12 in. high ?

10. Find the cost of digging a cellar 25 ft. by 10 ft., and 4 ft. deep, at 75 ct. a cubic yard.

11. Find the cost of carting away the excavated earth at 25 ct. a cartload, each cart holding $1\frac{1}{4}$ cu. yd.

12. The weight of ice is .92 of the weight of water. What is the weight of a cake of ice 3 ft. long, 2 ft. wide, and 1 ft. high ? (1 cu. ft. of water weighs $62\frac{1}{2}$ lb.)

13. One tank is 6 ft. high ; the radius of its base is 4 ft. How many gallons does it hold ?

14. Find the capacity of a tank that is as high as the one given in No. 13, but has a base whose radius is twice as great.

15. A box car is 33 ft. long, 8 ft. 8 in. wide, 7 ft. 6 in. high, inside measure.

(a) How many cubic feet in the car ?

(b) How many bushels will the car hold ?

350. Find the value for ? in each of the following :

	FIGURE	BASE	ALTITUDE	RADIUS	CIRCUM-FERENCE	AREA
1.	Rectangle	6 ft.	6 in.	—	—	?
2.	Rectangle	?	2 ft.	—	—	18 sq. ft.
3.	Square	?	?	—	—	256 sq. ft.
4.	Parallelogram	$1\frac{1}{2}$ ft.	9 in.	—	—	?
5.	Parallelogram	?	12 ft.	—	—	216 sq. ft.
6.	Parallelogram	24 ft.	?	—	—	384 sq. ft.
7.	Rectangle	?	$5\frac{1}{2}$ ft.	—	—	$41\frac{1}{2}$ sq. ft.
8.	Rectangle	$10\frac{1}{2}$ ft.	?	—	—	$26\frac{1}{2}$ sq. ft.
9.	Circle	—	—	7 ft.	?	?
10.	Circle	—	—	?	?	44 sq. in.
11.	Circle	—	—	?	22 in.	?
12.	Circle	—	—	$9\frac{1}{2}$ in.	?	?
13.	Triangle	$9\frac{1}{4}$ ft.	2 yd.	—	—	?
14.	Triangle	?	3 yd.	—	—	180 sq. ft.
15.	Triangle	2.8 ft.	?	—	—	9.8 sq. ft.
16.	Triangle	?	7 in.	—	—	9 sq. ft.
17.	Rectangle	12.6 ft.	2.7 yd.	—	—	?
18.	Triangle	5.6 ft.	1.3 yd.	—	—	?
19.	Square	?	?	—	—	42.25 sq. yd.
20.	Parallelogram	?	8.5 ft.	—	—	21.25 sq. ft.
21.	Circle	—	—	2.5 ft.	?	?
22.	Circle	—	—	?	?	36 sq. ft.
23.	Triangle	40.5 ft.	?	—	—	101.25 sq. ft.
24.	Square	?	?	—	—	21.16 sq. ft.
25.	Rectangle	?	8.9 ft.	—	—	30.26 sq. ft.

351. Find the value for ? in each of the following :

	FIGURE	PERIMETER OF BASE	ALTITUDE	SLANT HEIGHT	DIAMETER	CONVEX SURFACE
1.	Prism	16 sq. ft.	2 yd.	—	—	?
2.	Prism	24 sq. ft.	?	—	—	120 sq. ft.
3.	Cylinder	18.5 sq. ft.	$1\frac{1}{2}$ yd.	—	—	?

352. Find the value for ? in each of the following :

	FIGURE	AREA OF BASE	ALTITUDE	RADIUS	VOLUME
1.	Prism	48 sq. ft.	$2\frac{1}{2}$ yd.	—	?
2.	Cylinder	72 sq. in.	$1\frac{1}{2}$ ft.	—	?

MISCELLANEOUS PROBLEMS IN MENSURATION

353. 1. How many lots, each 25 ft. by 100 ft., can be made from a plot 300 ft. by 200 ft.? (Draw a diagram.)

2. The area of a rectangular plot is 1200 sq. ft. The base is three times as long as the altitude. Find the dimensions of the plot.

3. A rectangular field is 180 yd. long; the width is 40 % of the length. Find the area.

4. A triangular flag is made of two strips of felt of equal size. The base of the flag is 9 in.; the altitude of the flag is 18 in. How many square inches of felt in each strip?

5. A circular track is 140 ft. in diameter. How many times must a boy run around the track to travel a mile? ($R - 3\frac{1}{4}$.)

6. A bin 10 ft. long, 5 ft. wide, and 5 ft. deep is filled with wheat.

(a) How many bushels of wheat are in the bin?

(b) Find the value of the wheat at \$1.05 per bushel.

7. A rectangular stone prism is to have all its faces polished except the base. The base of the prism is 3 ft. 4 in. long and 2 ft. 10 in. wide. What will the work cost at 75 ct. a sq. ft.?

8. A cylindrical stone column is 12 ft. high; the area of the base is 12 sq. ft.

(a) Find the volume of the column in cubic yards.

(b) Find the weight of the column if one cubic yard weighs 42 lb.

(c) Find the area of the convex surface of the column.

(d) Find the area of the total surface of the column.

9. Draw to a convenient scale the plan of
(a) A baseball field.
(b) A lawn tennis court.
10. Make up and solve three problems about the baseball field and three problems about the lawn tennis court.

MISCELLANEOUS PROBLEMS—RELATED PROCESSES

Group I

354. 1. $24\frac{1}{2}$ yd. of cloth are bought for $\$36\frac{3}{4}$. Find the cost of one yard.
2. $30\frac{3}{4}$ yd. of cloth are bought for $\$1\frac{1}{5}$ per yard. Find the cost.
3. $32\frac{1}{2}$ yd. of cloth are bought for $\$40.36$. Find the cost of $16\frac{1}{4}$ yd.
4. 47 yd. of cloth are bought for $\$70\frac{1}{2}$. How many yards may be bought for $\$211\frac{1}{2}$?
5. Cloth bought at $\$1\frac{3}{4}$ per yard is sold for $\$2\frac{1}{4}$ per yard.
(a) Find the gain per cent on 1 yd.
(b) Find the gain per cent on 24 yd.
6. Cloth that cost $\$1\frac{3}{4}$ per yard is sold at a gain of 20 %. Find the selling price.
7. Cloth is sold at a gain of $33\frac{1}{3}\%$. If the selling price is $\$2$ per yard, what is the cost per yard?

Group II

355. Mr. Joseph Adams keeps a store at 214 East 23d St., New York. He orders the following goods from the American Hat Company, 130 Mercer St., New York:

- 4 doz. Derby Hats @ $\$30$ per dozen.
6 doz. Straw Hats @ $\$24$ per dozen.
10 doz. Boys' Caps @ $\$2$ per dozen.

1. Write, in full, the letter that Joseph Adams sends to the American Hat Company ordering the goods.

2. The American Hat Company answers Mr. Adams, thanking him for the order and stating that the goods will be sent within 5 days. Write the letter.

3. Write, in full, the bill that is sent with the goods. Fill out all the amounts, etc.

4. Mr. Adams paid as soon as the goods were received, and obtained discounts of 10 % and 2 %. How much did he pay ?

5. Write the check which Mr. Adams sent to the American Hat Company.

6. The American Hat Company deposits the check in the Corn Exchange Bank.

(a) Indorse the check so that if it is lost no one can collect the money.

(b) Make out the deposit slip.

7. Write the receipt sent by the American Hat Company to Mr. Adams.

Group III

356. 1. A merchant bought:

120 white shirts @ \$12 per dozen.

180 gray shirts @ \$18 per dozen.

Find the total cost of the shirts.

2. The white shirts were marked at a gain of 25 %. 75 % of them were sold at that price. How much was received for them ?

3. The rest of the white shirts were sold at a reduction of 10 % from the marked price. How much was received for them ?

4. The gray shirts were marked at a gain of 20 %. 60 % of them were sold at that price. How much was received for them ?

5. The rest of the gray shirts were sold at a reduction of 20 % from the marked price. How much was received for them ?

6. How much did the merchant gain or lose on the whole transaction ?

7. What per cent did he gain or lose on the whole transaction ?

Group IV

357. The lots shown on this diagram were sold at an auction sale. The prices are given below.

JEROME				AVE.							
209TH	57	56	55	54	53	52	51	50	49	48	47
	58										
	59										
	60										
210TH	45	44	43	42	41	40	39	38	37	36	35
	46										
	47										
	48										
ST.	31	34	33	32	31	30	29	28	27	26	25
	32										
	33										
	34										
BROADWAY											

31 - 25 × 100 (cor.)	\$11,800
32, 33, 34 - 25 × 100 (each)	4,800
35, 36 - 25 × 100 (each)	3,350
37, 38 - 25 × 100 (each)	3,850
39, 40 - 25 × 100 (each)	4,000
41 - 25 × 100 (each)	4,150
42, 43, 44 - 25 × 100 (each)	4,000
45 - 25 × 100 (cor.)	8,000
46, 47, 48 - 25 × 95 (each)	1,800
49 - 25 × 95 (cor.)	3,200
50, 51, 52, 53, 54 - 25 × 100 (each)	1,200
55, 56 - 25 × 100 (each)	1,000
57 - 25 × 95 (cor.)	2,000
58, 59, 60 - 25 × 95 (each)	1,800

1. Find the total area of the plot.
2. Find the average price per lot on Broadway.
3. Find the average price of all the lots.
4. The price of lot 31 is what per cent of the price of lot 32?
5. Find the area of lot 45.
6. Find the price per square foot of lot 45.
7. The buyer of lot 37 paid 30% in cash and allowed the rest to remain on mortgage. Find the face of the mortgage.
8. Find the semiannual interest on the mortgage at 5%.
9. Find the annual taxes on lot 50 at 2.15 per centum.
10. The buyer of lot 58 erected a house that cost \$5000. He sold the house and lot at a gain of 18%. How much did he receive for them? How much did he gain?

Group V

- 358.** 1. A train travels $20\frac{1}{4}$ mi. in an hour. How far does it go in $6\frac{3}{4}$ hr. at that rate?
2. If a train travels $112\frac{1}{2}$ mi. in $4\frac{1}{2}$ hr., how far does it go in one hour at that rate?
3. If a train travels $256\frac{1}{2}$ mi. in $8\frac{1}{2}$ hr., how far does it go in $4\frac{1}{4}$ hr. at that rate?
4. A train travels $98\frac{1}{8}$ mi. less in $5\frac{1}{2}$ hr. than in $8\frac{3}{4}$ hr. Find the number of miles it travels per hour.
5. A train traveled at the rate of 30 mi. per hour for $3\frac{1}{2}$ hr., then it increased its speed by $\frac{1}{3}$ of its former rate and traveled at the faster rate for $2\frac{1}{2}$ hr. How far did it travel in the entire time?

ECONOMY IN BUYING LARGE QUANTITIES, ETC.

359. 1. A family can buy eggs at 2 for 5 ct., or at 28 ct. a dozen. It uses 60 doz. a year. How much will be gained by buying the eggs by the dozen?

2. A dealer buys a case of eggs (30 doz.) for \$8.50. He sells them at the rate of 38 ct. a dozen.

(a) How much does he gain?

(b) What per cent does he gain?

3. A grocer buys a tub of butter (56 lb.) at 32 ct. a pound. He sells half the tub at 37 ct. a pound and the rest at 19 ct. a half pound.

(a) What per cent does he gain?

(b) How much does he gain on 8 tubs of butter?

4. A family buys butter at the rate of 10 ct. a quarter of a pound. If the family uses 12 lb. of butter a month, how much would be saved each month by buying it at 37 ct. a pound?

5. A grocer bought a barrel of flour (196 lb.) for \$5.80. He put the flour into $3\frac{1}{2}$ -lb. bags and sold it at 15 ct. a bag.

(a) How much did he gain on a barrel?

(b) What per cent did he gain?

6. Instead of buying a ton of coal for \$6.75 at one time, a family buys it in pails, each pail containing 25 lb., at 15 ct. a pail. How much does the family lose by buying the coal by the pail?

7. A grocer bought 24 bbl. of flour at \$5.60 a barrel. He sells $\frac{1}{2}$ of the quantity in $\frac{1}{8}$ -bbl. sacks at \$.85 a sack, and the rest in $\frac{1}{4}$ -bbl. sacks at \$1.65 a sack. How much does he gain on the transaction?

8. A family can buy sugar at the rate of $3\frac{1}{2}$ lb. for 19 ct., or 10 lb. for 54 ct.

(a) Which is the better rate?

(b) How much is saved by buying 70 lb. at the better rate?

9. A barrel of vinegar ($31\frac{1}{2}$ gal.) is bought at 40 ct. a gallon. It is sold at 5 ct. a quart.

(a) Find the per cent gain.

(b) Find the gain on 5 bbl.

10. A family can buy salt in 2-ct. bags containing 1 lb., or in 5-ct. bags containing 3 lb. If the family uses $4\frac{1}{2}$ lb. of salt a month, how much will it save in a year by buying 5-ct. bags?

11. Mr. Hunt buys 75 pr. of shoes at \$1.50 per pair and 100 pr. at \$2 per pair. He sells the cheaper shoes for \$1.75 per pair and the more expensive shoes at a gain of 50 ct. per pair.

(a) How much does he gain on the whole transaction?

(b) What per cent does he gain on the cheaper shoes?

12. I gained 15%, or \$9.75, by selling a sewing machine. How much did the machine cost?

13. I gained 22%, or \$32.20, by selling a rug. How much did I receive for the rug?

14. A clock was sold for \$81. This was a gain of 8% on the cost. Find the cost.

15. By selling a load of fruit for \$60, I lost 40%. Find the cost.

16. A druggist spends 85 ct. for a bottle of medicine. He sells it for \$1.25. What per cent does he gain?

17. A butcher buys 80 turkeys. The average weight is 15 lb. The turkeys cost 24 ct. a pound and are sold at a gain of $16\frac{2}{3}\%$.

(a) How much is received for all the turkeys?

(b) Find the per cent profit.

18. Oranges that were bought at the rate of 5 for 3 ct. are sold at the rate of 3 for 10 ct. Find the per cent of profit.

19. Newspapers that cost $\frac{1}{2}$ ct. each are sold at 1 ct. each. Find the per cent gain.

20. George had 50 ct. He bought newspapers at 2 for 1 ct. He sold the papers at 1 ct. each and spent all his money for more papers at 2 for 1 ct. He sold all of his second lot of papers at 1 ct. each.

(a) Find George's profit.

(b) Find the per cent of profit on the original capital.

21. What per cent is gained on lead pencils that are bought at the rate of 4 for 5 ct. and sold at the rate of 3 ct. each?

22. A tub of butter containing 52 lb. is bought for \$32 and sold at 40 ct. a pound. Find the per cent profit.

23. Mr. A sold 2 houses that cost \$22,500 each. On one he gained 34 %; on the second, he lost 34 %. Find the amount gained or lost in all.

24. Mr. A sold 2 houses at \$36,000 each. On one he gained 25 %; on the other, he lost 25 %. How much did he gain or lose in all?

25. A wagon that cost \$250 was marked at an advance of 50 %. It was sold at a discount of 25 % from the marked price. Find the loss or gain per cent.

26. A suit of clothes cost \$16. At what price should it be marked so that I may make a reduction of 25 % from the marked price and still gain 25 %?

APPENDIX I

TABLES FOR REFERENCE

Long Measure

Long measure, or linear measure, is used in measuring lengths or distances. The standard unit of length is the yard. The standard yard is a metal bar carefully preserved by the government in Washington. An exact copy is kept in each state capital.

Table of Long Measure

12 inches (in.)	= 1 foot (ft.)
3 feet	= 1 yard (yd.)
$5\frac{1}{2}$ yards	= 1 rod (rd.)
$16\frac{1}{2}$ feet	= 1 rod
320 rods	= 1 mile (mi.)
1760 yards	= 1 mile
5280 feet	= 1 mile

One eighth of a mile is called a furlong.

Surveyors, or measurers of land, use the following:

4 rods, or 66 feet, or 100 links (li.)	= 1 chain (ch.)
80 chains	= 1 mile

Square Measure

Square measure, or surface measure, is used in measuring surfaces. The standard unit of measurement is the square yard. It is a square surface one yard long and one yard wide.

Table of Square Measure

144 square inches (sq. in.)	= 1 square foot (sq. ft.)
9 square feet	= 1 square yard (sq. yd.)
30 $\frac{1}{4}$ square yards	= 1 square rod (sq. rd.)
160 square rods	= 1 acre (A.)
640 acres	= 1 square mile (sq. mi.)
36 square miles	= 1 township or section

The acre is used in measuring land. A square field 209 feet long and 209 feet wide measures about one acre.

Surveyors use the following:

16 square rods	= 1 square chain (sq. ch.)
10 square chains	= 1 acre

Miscellaneous Measures of Length or Surface

4 inches	= 1 hand (used in measuring the height of horses)
6 feet	= 1 fathom (used in measuring the depth of water)
1 $\frac{1}{8}$ miles	= 1 knot or nautical mile
100 square feet	= a square (of roof, pavement)

Cubic Measure

Cubic measure is used in measuring solids and other volumes. The standard unit is the cubic foot. It is equal to the volume of a cube one foot in length, one foot in width, and one foot in height or depth.

A body has three measurements or dimensions: length, breadth, and thickness or height.

A cube is a body whose length, breadth, and thickness are equal. It is bounded by six equal square surfaces.

A cubic foot is a cube each of whose dimensions is one foot.

Table of Cubic Measure

1728 cubic inches (cu. in.)	= 1 cubic foot (cu. ft.)
27 cubic feet	= 1 cubic yard (cu. yd.)
128 cubic feet of wood	= 1 cord
231 cubic inches (of liquid)	= 1 gallon
2150.4 cubic inches (of grain, etc.)	= 1 bushel (the standard Winchester bushel)

The standard bushel in England is the Imperial bushel; it contains 2218.192 cu. in.

A cord is a pile of wood 8 feet long, 4 feet wide, and 4 feet high.

Measure of Angles or Circular Measure

60 seconds (")	= 1 minute (')
60 minutes	= 1 degree (°)
90 degrees	= 1 right angle (rt. ∟) or 1 quadrant
360 angle degrees	= 4 right angles
360 arc degrees	= 1 circumference

Avoirdupois Weight

Avoirdupois weight is used in weighing grain, coal, and other bulky merchandise. The standard unit is the avoirdupois pound containing 7000 Troy grains.

Table of Avoirdupois Weight

16 ounces (oz.)	= 1 pound (lb.)
100 pounds	= 1 hundredweight (cwt.)
2000 pounds	= 1 ton (T.)
20 hundredweight	= 1 ton
2240 pounds	= 1 long ton
112 pounds	= 1 long hundredweight

Miscellaneous Weights

60 pounds (lb.)	= 1 bushel of potatoes
32 pounds	= 1 bushel of oats
60 pounds	= 1 bushel of wheat
56 pounds	= 1 bushel of corn (shelled)
56 pounds	= 1 bushel of rye
48 pounds	= 1 bushel of barley
80 pounds	= 1 bushel of coal
200 pounds	= 1 barrel of pork
196 pounds	= 1 barrel of flour
56 pounds	= 1 firkin of butter
62½ pounds	= 1 cubic foot of water

Troy Weight

Troy weight is used in weighing the precious metals such as gold and silver.

Table of Troy Weight

24 grains (gr.)	= 1 pennyweight (pwt. or dwt.)
20 pennyweights	= 1 ounce (oz.)
12 ounces	= 1 pound (lb.)
5760 grains	= 1 pound

An ounce Troy weighs 480 Troy grains.

Dry Measure

Dry measure is used in measuring dry articles such as grains, vegetables, and fruits. The standard unit is the Winchester bushel. It is cylindrical in form, 18½ inches in diameter, and 8 inches deep. It contains 2150.42 cubic inches, or 1¼ cubic feet.

Dry Measure		Liquid Measure *	
2 pints (pt.)	= 1 quart (qt.)	4 gills (gi.)	= 1 pint (pt.)
8 quarts	= 1 peck (pk.)	2 pints	= 1 quart (qt.)
4 pecks or } 32 quarts }	= 1 bushel (bu.)	4 quarts	= 1 gallon (gal.)

* The standard unit of liquid measure is the gallon, containing 231 cubic inches.

Time Measure

Time measure is used in measuring time. The standard unit of time is the day of 24 hours. It is the time required for the earth to make one complete rotation upon its axis.

The year consists of 365 days. It is the time required for the earth to make one complete revolution around the sun.

In telling the time of day, the hours before midday are called forenoon and those after midday are called afternoon. A.M. designates forenoon; P.M. afternoon. Thus, 7 A.M. means 7 o'clock in the morning, or forenoon; 4 P.M. means 4 o'clock in the afternoon; 12 M. indicates noon; 12 P.M. indicates midnight; 9.30 means 30 minutes after 9; 9.50 means 50 minutes after 9, or 10 minutes of or before 10.

Table of Time Measure

60 seconds (sec.)	= 1 minute (min.)
60 minutes	= 1 hour (hr.)
24 hours	= 1 day (da.)
365 days	= 1 year (yr.)
366 days	= 1 leap year.
100 years	= 1 century.

In examples involving reduction, 30 days are usually considered a month, and 12 months a year.

Thirty days has September.
April, June, and November, etc.

A **leap year** contains **366 days**; the extra day is added to February (Feb. 29). The system of leap year was devised to make up for the loss of nearly $\frac{1}{4}$ of a day each year, due to the difference between the actual length of a solar year (nearly $365\frac{1}{4}$ days) and the length of a calendar year (365 days).

Years indicated by numbers which are divisible by 4 without a remainder are leap years (*e.g.* 1912, 1916, 1920).

The only exception is that a leap year is not counted at the end of a century that is not exactly divisible by 400, *e.g.*, 1700, 1800, 1900 were not leap years; but 2000 will be a leap year.

Miscellaneous Tables

2 things	= 1 pair	20 things	= 1 score (sc.)
12 things	= 1 dozen (doz.)	24 sheets	= 1 quire (qr.)
12 dozen	} = 1 gross (gro.)	20 quires	} = 1 ream (rm.)
144 things		480 sheets	
12 gross, or	} = 1 great gross (gt. gro.)		
1728 things			

English Money

English money is the currency used in Great Britain and Ireland. The unit is the pound sterling (£), a gold coin, worth \$4.8665, or about \$4.87, in U. S. money. The shilling is worth a little more than 24 cents, and the penny is worth about 2 cents of our money.

Table of English Money

4 farthings (far.)	= 1 penny (d.)
12 pence	= 1 shilling (s.)
20 shillings	= 1 pound sterling (£)
21 shillings	= 1 guinea

The currency of systems of several other countries of commercial importance are shown in the following table:

Country	Standard	Divisions	U. S. Value
Germany	1 Mark	= 100 Pfennig	= \$.238
France	1 Franc	= 100 Centimes	= .193
Italy	1 Lira	= 100 Centesimi	= .193
Austria	1 Crown	= 100 Heller	= .203
Russia	1 Ruble	= 100 Kopecks	= .51

Table of Metric System

Metric Table of Length

10 millimeters (mm.)	= 1 centimeter (cm.)
10 centimeters	= 1 decimeter (dm.)
10 decimeters	= 1 meter (m.)
10 meters	= 1 dekameter (Dm.)
10 dekameters	= 1 hektometer (Hm.)
10 hektometers	= 1 kilometer (Km.)

Measurement of Surfaces

The **unit** of **surface measure** in the metric system is the **square meter**; it is the surface of a square, each side of which is one meter in length.

This diagram represents 1 square centimeter.

A **square meter** equals **1.196 square yards**.



Metric Table of Surface Measure

100 square millimeters (sq. mm.)	= 1 square centimeter (sq. cm.)
100 square centimeters	= 1 square decimeter (sq. dm.)
100 square decimeters	= 1 square meter (sq. m.).
100 square meters	= 1 square dekameter (sq. Dm.)
100 square dekameters	= 1 square hektometer (sq. Hm.)
100 square hektometers	= 1 square kilometer (sq. Km.)

The areas of walls, floors, ceilings, etc., are expressed in square meters; the areas of large surfaces, such as the areas of countries, are expressed in square kilometers (1 square kilometer equals nearly .4 square mile).

The **unit** of the metric table of **land measure** is the **are**, which is the area of a square dekameter; the area of an are, therefore, is

$$100 \times 1.196 \text{ sq. yd.} = 119.6 \text{ sq. yd.}$$

Metric Table of Land Measure

100 centares (ca.) = 1 are (a.)

100 ares = 1 hektare (nearly $2\frac{1}{2}$ acres) (Ha.)

Measurements of Volume

The unit of measurement of volume in the Metric System is the **cubic meter**; it is the volume of a cube each edge of which is one meter in length.

A **cubic meter** equals about 1.308 cubic yards, or $1\frac{1}{8}$ cubic yards.

Metric Table of Volume

1000 cubic millimeters = 1 **cubic centimeter**

1000 cubic centimeters = 1 **cubic decimeter**

1000 cubic decimeters = 1 **cubic meter**

The unit of the metric table of wood measure is the **stere**, which is a cubic meter; the stere is the only unit used.

Measures of Capacity

The unit of measurement of **capacity** in the metric system for both dry and liquid measures is the **liter**, which is the volume of a **cubic decimeter**.

A liter is about 1 quart; it is equal to 1.057 qt. (liquid measure) and .908 qt. (dry measure).

Metric Table of Dry and Liquid Measures

10 millimeters = 1 centiliter

10 centiliters = 1 deciliter

10 deciliters = 1 **liter** (l.)

10 liters = 1 dekaliter

10 dekaliters = 1 **hektoliter**

Small quantities, such as are purchased in stores, are measured by the liter; large quantities, those that are measured by the bushel in our system, are measured by the hektoliter (nearly 2.84 bushels).

Measures of Weight

The **unit** of measurement of **weight** in the Metric System is the **gram**; a gram is the weight of 1 cubic centimeter of water at its greatest density. Our nickel (5-cent piece) weighs about 5 grams.

A **kilogram** or **kilo** is the weight of 1000 cubic centimeters of water, or 1 liter of water. A **kilogram** weighs **2.2046 pounds** avoirdupois or 2.2 pounds.

Metric Table of Measures of Weight

10 milligrams (mg.)	= 1 centigram
10 centigrams	= 1 decigram
10 decigrams	= 1 gram (gr.)
10 grams	= 1 dekagram
10 dekagrams	= 1 hectogram
10 hectograms	= 1 kilogram (kilo) = 2.2046 lb.
1000 kilograms	= 1 metric ton.

The **gram** is a light weight (about $\frac{1}{7\frac{7}{8}}$ ounce avoirdupois); it is used in weighing precious metals, medicines, etc.; the **kilogram** is used in weighing groceries as ordinarily bought and sold; the **metric ton** (about 2205 pounds) is used in weighing heavy or bulky articles, such as coal, iron ore, etc.

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